

Neutron–proton radiative capture, photo–magnetic and anti–neutrino disintegration of the deuteron in the relativistic field theory model of the deuteron

A. N. Ivanov ^{*†}, H. Oberhummer [†], N. I. Troitskaya [‡], M. Faber [§]

February 9, 2008

*Institut für Kernphysik, Technische Universität Wien,
Wiedner Hauptstr. 8-10, A-1040 Vienna, Austria*

Abstract

The cross sections for the M1–capture $n + p \rightarrow D + \gamma$, the photo–magnetic and anti–neutrino disintegration of the deuteron are evaluated in the relativistic field theory model of the deuteron (RFMD). The cross section for M1–capture is evaluated by taking into account the contributions of chiral one–meson loop corrections and the $\Delta(1232)$ resonance. The cross sections for the photo–magnetic and anti–neutrino disintegration of the deuteron are evaluated by accounting for final–state interaction of the nucleon pair in the 1S_0 –state. The amplitudes of low–energy elastic np and nn scattering contributing to these processes are obtained in terms of the S–wave scattering lengths and the effective ranges. This relaxes substantially the statement by Bahcall and Kamionkowski (Nucl. Phys. A625 (1997) 893) that the RFMD is unable to describe a non–zero effective range for low–energy elastic nucleon–nucleon scattering. The cross sections for the anti–neutrino disintegration of the deuteron averaged over the anti–neutrino energy spectrum agree good with experimental data.

PACS: 11.10.Ef, 13.75.Cs, 14.20.Dh, 21.30.Fe

Keywords: field theory, QCD, deuteron, disintegration, photon, anti–neutrino, fusion

^{*}E–mail: ivanov@kph.tuwien.ac.at, Tel.: +43–1–58801–14261, Fax: +43–1–58801–14299

[†]E–mail: ohu@kph.tuwien.ac.at, Tel.: +43–1–58801–14251, Fax: +43–1–58801–14299

[‡]Permanent Address: State Technical University, Department of Nuclear Physics, 195251 St. Petersburg, Russian Federation

[§]E–mail: faber@kph.tuwien.ac.at, Tel.: +43–1–58801–14261, Fax: +43–1–58801–14299

1 Introduction

As we have shown in Ref. [1] the relativistic field theory model of the deuteron (RFMD) [2–6] is motivated by QCD. The deuteron appears as a neutron–proton collective excitation – a Cooper np–pair induced by a phenomenological local four–nucleon interaction in the nuclear phase of QCD. Strong low–energy interactions of the deuteron coupled to itself and other particles are described in terms of one–nucleon loop exchanges. The one–nucleon loop exchanges allow to transfer nuclear flavours from an initial to a final nuclear state by a minimal way and to take into account contributions of nucleon–loop anomalies determined completely by one–nucleon loop diagrams. The dominance of contributions of nucleon–loop anomalies has been justified in the large N_C expansion, where N_C is the number of quark colours [1]¹.

In this paper we apply the RFMD to the evaluation of the cross sections for the radiative M1–capture $n + p \rightarrow D + \gamma$ for thermal neutrons caused by the $^1S_0 \rightarrow ^3S_1$ transition, the photo–magnetic $\gamma + D \rightarrow n + p$ and anti–neutrino disintegration of the deuteron caused by charged $\bar{\nu}_e + D \rightarrow e^+ + n + n$ and neutral $\bar{\nu}_e + D \rightarrow \bar{\nu}_e + n + p$ weak currents. We would like to emphasize that the main goal of the paper is to show that: 1) Chiral perturbation theory can be incorporated in the RFMD, and 2) the amplitudes of low–energy elastic nucleon–nucleon scattering contributing to the reactions of the photo–magnetic and anti–neutrino disintegration of the deuteron can be described in the RFMD in agreement with low–energy nuclear phenomenology.

As has been found in Refs.[4–6] the cross section for the M1–capture calculated in the RFMD in the tree–meson approximation $\sigma^{\text{np}} = 276 \text{ mb}$ differs from the experimental data $\sigma_{\text{exp}}^{\text{np}} = (334.2 \pm 0.5) \text{ mb}$ [7] by about 17% of the experimental value. However, as has been shown in Refs. [8] contributions of chiral meson–loop corrections play an important role for the correct description of the process $n + p \rightarrow D + \gamma$. The evaluation of these corrections demands the use of Chiral perturbation theory.

For the evaluation of chiral one–meson loop corrections in the RFMD we use Chiral perturbation theory at the quark level $(\text{CHPT})_q$ with a linear realization of chiral $U(3) \times U(3)$ symmetry developed in Refs.[9,10]. The main chiral one–meson loop corrections are induced by following virtual meson transitions $\pi \rightarrow a_1 \gamma$, $a_1 \rightarrow \pi \gamma$, $\pi \rightarrow (\omega, \rho) \gamma$, $(\omega, \rho) \rightarrow \pi \gamma$, $\sigma \rightarrow (\omega, \rho) \gamma$ and $(\omega, \rho) \rightarrow \sigma \gamma$, where σ is a scalar partner of pions under chiral $SU(2) \times SU(2)$ symmetry [9,10].

However, as has been stated by Riska and Brown [11] (see also [17]) for correct description of the amplitude of the neutron–proton radiative capture one needs to take into account the contribution of the $\Delta(1232)$ resonance. In the RFMD the contribution of the $\Delta(1232)$ resonance has been considered in Ref.[12] by example of the evaluation of the S–wave scattering length of low–energy elastic πD scattering.

At threshold the amplitude of the photo–magnetic disintegration of the deuteron is related to the amplitude of the M1–capture. In order to evaluate the amplitude of $\gamma + D$

¹In Ref.[6] we have considered a modified version of the RFMD which is not well defined due to a violation of Lorentz invariance of the effective four–nucleon interaction describing $N + N \rightarrow N + N$ transitions. This violation has turned out to be incompatible with a dominance of one–nucleon loop anomalies which are Lorentz covariant. Thereby, the astrophysical factor for the solar proton burning calculated in Ref.[5] and enhanced by a factor of 1.4 with respect to the recommended value (E. G. Adelberger *et al.*, Rev. Mod. Phys. 70 (1998) 1265) is not good established.

$\rightarrow n + p$ for the energy region far from threshold we take into account the contributions of the np interaction in the final state. For this aim we sum up an infinite series of one-nucleon loop diagrams and evaluate the result of the summation in leading order in the large N_C expansion [1]. This gives the amplitude of low-energy elastic np scattering contributing to the amplitude of $\gamma + D \rightarrow n + p$ defined by the S-wave scattering length and the effective range.

The developed technique we apply to the evaluation of the cross section for the anti-neutrino disintegration of the deuteron $\bar{\nu}_e + D \rightarrow e^+ + n + n$ and $\bar{\nu}_e + D \rightarrow \bar{\nu}_e + n + p$. The reaction $\bar{\nu}_e + D \rightarrow e^+ + n + n$ is caused by charged weak current and valued, in the sense of charge independence of weak interaction strength, to be equivalent to the observation of the reaction of the solar proton burning, or pp fusion, $p + p \rightarrow D + e^+ + \nu_e$ in the terrestrial laboratories [13]. We compare the theoretical cross sections with recent experimental data given by the Reines's experimental group [14].

For the description of low-energy transitions $N + N \rightarrow N + N$ in the reactions $n + p \rightarrow D + \gamma$, $\gamma + D \rightarrow n + p$, $\bar{\nu}_e + D \rightarrow e^+ + n + n$, $\bar{\nu}_e + D \rightarrow \bar{\nu}_e + n + p$ and $p + p \rightarrow D + e^+ + \nu_e$, where nucleons are in the 1S_0 -state, we apply the effective local four-nucleon interactions [2–5]:

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\text{NN} \rightarrow \text{NN}}(x) = & G_{\pi\text{NN}} \{ [\bar{n}(x) \gamma_\mu \gamma^5 p^c(x)] [\bar{p}^c(x) \gamma^\mu \gamma^5 n(x)] \\ & + \frac{1}{2} [\bar{n}(x) \gamma_\mu \gamma^5 n^c(x)] [\bar{n}^c(x) \gamma^\mu \gamma^5 n(x)] + \frac{1}{2} [\bar{p}(x) \gamma_\mu \gamma^5 p^c(x)] [\bar{p}^c(x) \gamma^\mu \gamma^5 p(x)] \\ & + (\gamma_\mu \gamma^5 \otimes \gamma^\mu \gamma^5 \rightarrow \gamma^5 \otimes \gamma^5) \}, \end{aligned} \quad (1.1)$$

where $n(x)$ and $p(x)$ are the operators of the neutron and the proton interpolating fields, $n^c(x) = C \bar{n}^T(x)$ and so on, then C is a charge conjugation matrix and T is a transposition. The effective coupling constant $G_{\pi\text{NN}}$ is defined by [3–5]

$$G_{\pi\text{NN}} = \frac{g_{\pi\text{NN}}^2}{4M_\pi^2} - \frac{2\pi a_{\text{np}}}{M_N} = 3.27 \times 10^{-3} \text{ MeV}^{-2}, \quad (1.2)$$

where $g_{\pi\text{NN}} = 13.4$ is the coupling constant of the πNN interaction, $M_\pi = 135 \text{ MeV}$ is the pion mass, $M_p = M_n = M_N = 940 \text{ MeV}$ is the mass of the proton and the neutron neglecting the electromagnetic mass difference, which is taken into account only for the calculation of the phase volumes of the final states of the reactions $\bar{\nu}_e + D \rightarrow e^+ + n + n$, $\bar{\nu}_e + D \rightarrow \bar{\nu}_e + n + p$ and $p + p \rightarrow D + e^+ + \nu_e$, and $a_{\text{np}} = (-23.75 \pm 0.01) \text{ fm}$ is the S-wave scattering length of np scattering in the 1S_0 -state.

The first term in the effective coupling constant $G_{\pi\text{NN}}$ comes from the one-pion exchange for the squared momenta transfer $-q^2$ much less than the squared pion mass $-q^2 \ll M_\pi^2$ and the subsequent Fierz transformation of nucleon fields (see Appendix B of Ref. [6]). We should emphasize that due to Fierz transformation the effective local four-nucleon interaction caused by the one-pion exchange contains a few contributions with different spinorial structure, we have taken into account only those terms which contribute to the 1S_0 -state of the NN system. The second term in Eq.(1.2) is a phenomenological one representing a collective contribution caused by the integration over heavy meson degrees of freedom [5,6]. This term is taken in the form used in the Effective Field Theory (EFT) approach [15,16] for the description of low-energy elastic NN scattering. The effective interaction Eq. (1.1) is written in isotopically invariant form, and the coupling constant

$G_{\pi\text{NN}}$ can be never equal to zero at $a_{\text{np}} \neq 0$ due to a negative value of a_{np} imposed by nuclear forces, $a_{\text{np}} < 0$ [17]. Note that the contribution of the phenomenological part to the effective coupling constant $G_{\pi\text{NN}}$ makes up less than 33%.

In the low-energy limit the effective local four-nucleon interaction Eq.(1.1) vanishes due to the reduction

$$[\bar{N}(x)\gamma_\mu\gamma^5N^c(x)][\bar{N}^c(x)\gamma^\mu\gamma^5N(x)] \rightarrow -[\bar{N}(x)\gamma^5N^c(x)][\bar{N}^c(x)\gamma^5N(x)], \quad (1.3)$$

where $N(x)$ is the neutron or the proton interpolating field. Such a vanishing of the one-pion exchange contribution to the NN potential is well-known in the EFT approach [15,16] and the potential model approach (PMA) [17]. In power counting [15,16] the interaction induced by the one-pion exchange is of order $O(k^2)$, where k is a relative momentum of the NN system. The former is due the Dirac matrix γ^5 which leads to the interaction between small components of Dirac bispinors of nucleon wave functions.

In the one-nucleon loop exchange approach the contributions of the interactions $[\bar{N}(x)\gamma_\mu\gamma^5N^c(x)][\bar{N}^c(x)\gamma^\mu\gamma^5N(x)]$ and $[\bar{N}(x)\gamma^5N^c(x)][\bar{N}^c(x)\gamma^5N(x)]$ to amplitudes of nuclear processes are different and do not cancel each other in the low-energy limit due to the dominance of nucleon-loop anomalies [1]. This provides the interaction between large components of Dirac bispinors of nucleon wave functions.

The paper is organized as follows. In Sect.2 we evaluate the contribution of chiral one-meson loop corrections to the amplitude of the neutron-proton radiative capture and the cross section for the neutron-proton radiative capture. In Sect.3 we include the contribution of the $\Delta(1232)$ resonance and analyse the total cross section for the neutron-proton radiative capture for thermal neutrons and compare it with experimental data. In Sect.4 we evaluate the cross section for $\gamma + \text{D} \rightarrow \text{n} + \text{p}$ for energies far from threshold. The contribution of low-energy elastic np scattering to the amplitude of the process $\gamma + \text{D} \rightarrow \text{n} + \text{p}$ is evaluated in agreement with low-energy nuclear phenomenology. This relaxes substantially the statement by Bahcall and Kamionkowski [18] that in the RFMD due to the local four-nucleon interaction Eq.(1.1) one cannot describe low-energy elastic NN scattering in agreement with low-energy nuclear phenomenology. However, the problem of the description of low-energy elastic pp scattering accounting for the Coulomb repulsion still remains. In Sects.5 and 6 we evaluate the cross sections for the anti-neutrino disintegration of the deuteron caused by charged $\bar{\nu}_e + \text{D} \rightarrow e^+ + \text{n} + \text{n}$ and neutral $\bar{\nu}_e + \text{D} \rightarrow \bar{\nu}_e + \text{n} + \text{p}$ weak currents and average them over the anti-neutrino energy spectrum. The average values of the cross sections agree good with experimental data. In the Conclusion we discuss the obtained results.

2 Neutron-proton radiative capture

At low energies the neutron-proton radiative capture $\text{n} + \text{p} \rightarrow \text{D} + \gamma$ runs through the magnetic dipole transition $^1\text{S}_0 \rightarrow ^3\text{S}_1$, the M1-capture. In the RFMD the amplitude of the M1-capture calculated in the tree-meson approximation reads [3–6]²

$$\mathcal{M}(\text{n} + \text{p} \rightarrow \text{D} + \gamma) = e(\mu_{\text{p}} - \mu_{\text{n}}) \frac{5g_{\text{V}}}{8\pi^2} G_{\pi\text{NN}} (\vec{q} \times \vec{e}^*(\vec{q})) \cdot \vec{e}^*(\vec{k}_{\text{D}}) [\bar{u}^c(p_2)\gamma^5 u(p_1)], \quad (2.1)$$

²For the details of the calculation we relegate readers to Appendix F of Ref. [6].

where e is the proton electric charge, $\mu_p = 2.793$ and $\mu_n = -1.913$ are the magnetic dipole moments of the proton and the neutron, respectively, measured in nuclear magnetons, g_V is a phenomenological coupling constant of the RFMD related to the electric quadrupole moment of the deuteron $Q_D = 0.286 \text{ fm}^2$ [3]: $g_V^2 = 2\pi^2 Q_D M_N^2$; \vec{q} and \vec{k}_D are 3-momenta of the photon and the deuteron, and $\vec{e}^*(\vec{q})$ and $\vec{e}^*(\vec{k}_D)$ – the polarization vectors of them; $\bar{u}^c(p_2)$ and $u(p_1)$ are the Dirac bispinors of the neutron and the proton.

The cross section for the M1-capture calculated in the tree-meson approximation is then defined [3–6]:

$$\sigma^{\text{np}}(k) = \frac{1}{v} (\mu_p - \mu_n)^2 \frac{25}{64} \frac{\alpha}{\pi^2} Q_D G_{\pi\text{NN}}^2 M_N \epsilon_D^3 = 276 \text{ mb}, \quad (2.2)$$

where k is a relative momentum of the np system. The numerical value has been computed for $k = 0$, $\epsilon_D = 2.225 \text{ MeV}$ and $v = 7.34 \times 10^{-6}$ (the absolute value $v = 2.2 \times 10^5 \text{ cm s}^{-1}$), the laboratory velocity of the neutron. The theoretical value $\sigma^{\text{np}}(k) = 276 \text{ mb}$ agrees within an accuracy better than 10% with the theoretical value [17]

$$\sigma_{\text{PMA}}^{\text{np}}(k) = (302.5 \pm 4) \text{ mb} \quad (2.3)$$

calculated in the PMA for the pure M1 transition. In comparison with the experimental data [7]

$$\sigma_{\text{exp}}^{\text{np}} = (334.2 \pm 0.5) \text{ mb} \quad (2.4)$$

the theoretical value Eq.(2.2) obtained in the RFMD is less by 17% of the experimental one.

However, as has been shown in Refs. [8] chiral meson-loop corrections play an important role for the correct description of the low-energy process $n + p \rightarrow D + \gamma$ for thermal neutrons. The evaluation of chiral meson-loop corrections in the RFMD we use (CHPT) $_q$ developed in Refs.[9,10]. Below we consider the contributions of chiral one-meson loop corrections induced by the virtual meson transitions $\pi \rightarrow a_1 \gamma$, $a_1 \rightarrow \pi \gamma$, $\pi \rightarrow (\omega, \rho) \gamma$, $(\omega, \rho) \rightarrow \pi \gamma$, $\sigma \rightarrow (\omega, \rho) \gamma$ and $(\omega, \rho) \rightarrow \sigma \gamma$, where σ is a scalar partner of pions under chiral $SU(2) \times SU(2)$ transformations in (CHPT) $_q$ with a linear realization of chiral $U(3) \times U(3)$ symmetry [9,10].

The effective Lagrangians $\delta \mathcal{L}_{\text{eff}}^{\text{pp}\gamma}(x)$ and $\delta \mathcal{L}_{\text{eff}}^{\text{nn}\gamma}(x)$, caused by the virtual meson transitions $\pi \rightarrow a_1 \gamma$, $a_1 \rightarrow \pi \gamma$, $\pi \rightarrow (\omega, \rho) \gamma$, $(\omega, \rho) \rightarrow \pi \gamma$, $\sigma \rightarrow (\omega, \rho) \gamma$ and $(\omega, \rho) \rightarrow \sigma \gamma$, we evaluate in leading order in the large N_C expansion [1]. The results of the evaluation contain divergent contributions. In order to remove these divergences we apply the renormalization procedure developed in (CHPT) $_q$ for the evaluation of chiral meson-loop corrections (see *Ivanov* in Refs. [9]). Since the renormalized expressions should vanish in the chiral limit $M_\pi \rightarrow 0$ [9], only the virtual meson transitions with intermediate π -meson give non-trivial contributions. The contributions of the virtual meson transitions with intermediate σ -meson are found finite in the chiral limit and subtracted according to the renormalization procedure [9]. Such a cancellation of the σ -meson contributions in the one-meson loop approximation agrees with Chiral perturbation theory using a non-linear realization of chiral symmetry, where σ -meson like exchanges can appear only in two-meson loop approximation. Then, the sum of the contributions of the virtual meson

transitions $\pi^- \rightarrow \rho^- \gamma$, $\pi^0 \rightarrow \rho^0 \gamma$ and $\pi^0 \rightarrow \omega \gamma$ to the effective coupling $nn\gamma$ is equal to zero. As a result the effective Lagrangians $\delta\mathcal{L}_{\text{eff}}^{\text{pp}\gamma}(x)$ and $\delta\mathcal{L}_{\text{eff}}^{\text{nn}\gamma}(x)$ are given by

$$\begin{aligned}\delta\mathcal{L}_{\text{eff}}^{\text{pp}\gamma}(x) &= \frac{ie}{4M_N} \left[g_A g_{\pi NN} \frac{\alpha_\rho}{16\pi^3} \frac{M_N}{F_\pi} M_\pi^2 J_{\pi a_1 N} + g_{\pi NN} \frac{N_C \alpha_\rho}{16\pi^3} \frac{M_N}{F_\pi} M_\pi^2 J_{\pi VN} \right] \\ &\quad \times [\bar{p}(x) \sigma_{\mu\nu} p(x)] F^{\mu\nu}(x), \\ \delta\mathcal{L}_{\text{eff}}^{\text{nn}\gamma}(x) &= \frac{ie}{4M_N} \left[-g_A g_{\pi NN} \frac{\alpha_\rho}{16\pi^3} \frac{M_N}{F_\pi} M_\pi^2 J_{\pi a_1 N} \right] [\bar{n}(x) \sigma_{\mu\nu} n(x)] F^{\mu\nu}(x),\end{aligned}\quad (2.5)$$

where $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ is the electromagnetic field strength, $\alpha_\rho = g_\rho^2/4\pi = 2.91$ is the effective coupling constant of the $\rho \rightarrow \pi\pi$ decay, $F_\pi = 92.4 \text{ MeV}$ is the leptonic coupling constant of pions, and $g_A = 1.267$ [19]. Then, $J_{\pi a_1 N}$ and $J_{\pi VN}$ are the momentum integrals determined by

$$\begin{aligned}J_{\pi a_1 N} &= \int \frac{d^4 p}{\pi^2} \frac{1}{(M_\pi^2 + p^2)(M_{a_1}^2 + p^2)(M_N^2 + p^2)} = 0.017 M_\pi^{-2}, \\ J_{\pi VN} &= \int \frac{d^4 p}{\pi^2} \frac{1}{(M_\pi^2 + p^2)(M_V^2 + p^2)(M_N^2 + p^2)} = 0.024 M_\pi^{-2},\end{aligned}\quad (2.6)$$

where p is Euclidean 4-momentum, $M_V = M_\rho = M_\omega = 770 \text{ MeV}$ [19] and $M_{a_1} = \sqrt{2} M_\rho$ [9].

At $N_C = 3$ the cross section for the M1-capture accounting for the contribution of the effective interaction Eq.(2.5) amounts to

$$\begin{aligned}\sigma^{\text{np}}(k) &= \frac{1}{v} (\mu_p - \mu_n)^2 \frac{25}{64} \frac{\alpha}{\pi^2} Q_D G_{\pi NN}^2 M_N \varepsilon_D^3 \\ &\quad \times \left[1 + \frac{g_{\pi NN}^2}{\mu_p - \mu_n} \frac{M_\pi^2}{8\pi^2} \frac{\alpha_\rho}{\pi} \left(J_{\pi a_1 N} + \frac{3}{2g_A} J_{\pi VN} \right) \right]^2 = 287.2 \text{ mb},\end{aligned}\quad (2.7)$$

where we have used the relation $g_{\pi NN} \simeq g_A M_N/F_\pi$. The theoretical value of the cross section for the neutron-proton radiative capture given by Eq.(2.7) differs from the experimental one by about 14%. This discrepancy we describe by taking into account the contribution of the $\Delta(1232)$ resonance.

3 $\Delta(1232)$ resonance

In our consideration the $\Delta(1232)$ resonance is the Rarita-Schwinger field [20] $\Delta_\mu^a(x)$, the isotypical index a runs over $a = 1, 2, 3$, having the following free Lagrangian [21,22]:

$$\mathcal{L}_{\text{kin}}^\Delta(x) = \bar{\Delta}_\mu^a(x) [-(i\gamma^\alpha \partial_\alpha - M_\Delta) g^{\mu\nu} + \frac{1}{4} \gamma^\mu \gamma^\beta (i\gamma^\alpha \partial_\alpha - M_\Delta) \gamma_\beta \gamma^\nu] \Delta_\nu^a(x), \quad (3.1)$$

where $M_\Delta = 1232 \text{ MeV}$ is the mass of the Δ resonance field $\Delta_\mu^a(x)$. In terms of the eigenstates of the electric charge operator the fields $\Delta_\mu^a(x)$ are given by [12,21,22]

$$\begin{aligned}\Delta_\mu^1(x) &= \frac{1}{\sqrt{2}} \begin{pmatrix} \Delta_\mu^{++}(x) - \Delta_\mu^0(x)/\sqrt{3} \\ \Delta_\mu^+(x)/\sqrt{3} - \Delta_\mu^-(x) \end{pmatrix}, \quad \Delta_\mu^2(x) = \frac{i}{\sqrt{2}} \begin{pmatrix} \Delta_\mu^{++}(x) + \Delta_\mu^0(x)/\sqrt{3} \\ \Delta_\mu^+(x)/\sqrt{3} + \Delta_\mu^-(x) \end{pmatrix}, \\ \Delta_\mu^3(x) &= -\sqrt{\frac{2}{3}} \begin{pmatrix} \Delta_\mu^+(x) \\ \Delta_\mu^0(x) \end{pmatrix}.\end{aligned}\quad (3.2)$$

The fields $\Delta_\mu^a(x)$ obey the subsidiary constraints: $\partial^\mu \Delta_\mu^a(x) = \gamma^\mu \Delta_\mu^a(x) = 0$ [20–22]. The Green function of the free Δ -field is determined

$$\langle 0 | T(\Delta_\mu(x_1) \bar{\Delta}_\nu(x_2)) | 0 \rangle = -i S_{\mu\nu}(x_1 - x_2). \quad (3.3)$$

In the momentum representation $S_{\mu\nu}(x)$ reads [12,21,22]:

$$S_{\mu\nu}(p) = \frac{1}{M_\Delta - \hat{p}} \left(-g_{\mu\nu} + \frac{1}{3} \gamma_\mu \gamma_\nu + \frac{1}{3} \frac{\gamma_\mu p_\nu - \gamma_\nu p_\mu}{M_\Delta} + \frac{2}{3} \frac{p_\mu p_\nu}{M_\Delta^2} \right). \quad (3.4)$$

The most general form of the $\pi N \Delta$ - interaction compatible with the requirements of chiral symmetry reads [21]:

$$\begin{aligned} \mathcal{L}_{\pi N \Delta}(x) &= \frac{g_{\pi N \Delta}}{2M_N} \bar{\Delta}_\omega^a(x) \Theta^{\omega\varphi} N(x) \partial_\varphi \pi^a(x) + \text{h.c.} = \\ &= \frac{g_{\pi N \Delta}}{\sqrt{6} M_N} \left[\frac{1}{\sqrt{2}} \bar{\Delta}_\omega^+(x) \Theta^{\omega\varphi} n(x) \partial_\varphi \pi^+(x) - \frac{1}{\sqrt{2}} \bar{\Delta}_\omega^0(x) \Theta^{\omega\varphi} p(x) \partial_\varphi \pi^-(x) \right. \\ &\quad \left. - \bar{\Delta}_\omega^+(x) \Theta^{\omega\varphi} p(x) \partial_\varphi \pi^0(x) - \bar{\Delta}_\omega^0(x) \Theta^{\omega\varphi} p(x) \partial_\varphi \pi^0(x) + \dots \right]. \end{aligned} \quad (3.5)$$

The nucleon field $N(x)$ is the isotopical doublet with the components $N(x) = (p(x), n(x))$, and $\pi^a(x)$ is the pion field with the components $\pi^1(x) = (\pi^-(x) + \pi^+(x))/\sqrt{2}$, $\pi^2(x) = (\pi^-(x) - \pi^+(x))/i\sqrt{2}$ and $\pi^3(x) = \pi^0(x)$. The tensor $\Theta^{\omega\varphi}$ is given in Ref. [21]: $\Theta^{\omega\varphi} = g^{\omega\varphi} - (Z + 1/2) \gamma^\omega \gamma^\varphi$, where the parameter Z is arbitrary. There is no consensus on the exact value of Z . From theoretical point of view $Z = 1/2$ is preferred [21]. Phenomenological studies give only the bound $|Z| \leq 1/2$ [23]. The empirical value of the coupling constant $g_{\pi N \Delta}$ relative to the coupling constant $g_{\pi N N}$ is $g_{\pi N \Delta} = 2.12 g_{\pi N N}$ [24].

Assuming that the transition $\Delta \rightarrow N + \gamma$ is primarily a magnetic one the effective Lagrangian describing the $\Delta \rightarrow N + \gamma$ decays can be determined as [25,26]:

$$\begin{aligned} \mathcal{L}_{\gamma N \Delta}(x) &= ie \frac{g_{\gamma N \Delta}}{2M_N} \bar{N}(x) \gamma_\alpha \gamma^5 \Delta_\beta^3(x) F^{\beta\alpha}(x) + \text{h.c.} = \\ &= -\frac{ie}{\sqrt{6}} \frac{g_{\pi N \Delta}}{M_N} [\bar{p}(x) \gamma_\alpha \gamma^5 \Delta_\beta^+(x) + \bar{n}(x) \gamma_\alpha \gamma^5 \Delta_\beta^0(x)] F^{\beta\alpha}(x) + \text{h.c.}, \end{aligned} \quad (3.6)$$

where $F^{\alpha\beta}(x) = \partial^\alpha A^\beta(x) - \partial^\beta A^\alpha(x)$ and $A^\alpha(x)$ is the operator of the photon field. The empirical value of the coupling constant $g_{\gamma N \Delta}$ relative to the coupling constant $g_{\pi N N}$ is $g_{\gamma N \Delta} = 0.32 g_{\pi N N}$ [27].

For the calculation of the amplitude of the neutron-proton radiative capture in the RFMD we have to calculate the effective Lagrangian describing the $n + p \rightarrow \Delta + N$ transitions. Following the general procedure expounded in Ref. [3] we obtain:

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{np \rightarrow \Delta N}(x) &= -\frac{i}{\sqrt{6}} \frac{g_{\pi N \Delta}}{M_N} \frac{g_{\pi N N}}{4M_\pi^2} \int d^4 z \frac{\partial}{\partial z_\varphi} \delta^{(4)}(z - x) \{ [\bar{\Delta}_\omega^+(z) \Theta^\omega_\varphi n^c(x)] \\ &\quad \times [\bar{n}^c(z) \gamma^5 p(x) + \bar{n}^c(x) \gamma^5 p(z)] - [\bar{\Delta}_\omega^0(z) \Theta^\omega_\varphi p^c(x)] [\bar{n}^c(z) \gamma^5 p(x) + \bar{n}^c(x) \gamma^5 p(z)] \\ &\quad + 1 \otimes \gamma^5 \rightarrow -\gamma_\nu \otimes \gamma^\nu \gamma^5 \}. \end{aligned} \quad (3.7)$$

Using then the phenomenological Lagrangian

$$\mathcal{L}_{npD}(x) = -ig_V [\bar{p}^c(x) \gamma^\mu n(x) - \bar{n}^c(x) \gamma^\mu p(x)] D_\mu^\dagger(x) \quad (3.8)$$

the effective Lagrangian describing the contribution of the Δ resonance to the amplitude of the transition $n + p \rightarrow D + \gamma$ is defined [5]

$$\begin{aligned}
\int d^4x \mathcal{L}_{np \rightarrow \Delta N \rightarrow D\gamma}(x) &= - \int d^4x_1 d^4x_2 d^4x_3 < T(\mathcal{L}_{\text{eff}}^{np \rightarrow \Delta N}(x_1) \mathcal{L}_{npD}(x_2) \mathcal{L}_{\gamma N\Delta}(x_3)) > = \\
&= -\frac{i}{6} \frac{eg_V}{M_N^2} \frac{g_{\pi N\Delta}}{g_{\pi NN}} \frac{g_{\gamma N\Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} \int d^4x_1 d^4x_2 d^4x_3 \int d^4z \frac{\partial}{\partial z_\varphi} \delta^{(4)}(z - x_1) \\
&\times T([p^c(x_1) \gamma^5 n(z) + \bar{p}^c(z) \gamma^5 n(x_1)] D_\mu^\dagger(x_2) F^{\alpha\beta}(x_3)) \\
&\times \left\{ < 0 | T([\bar{\Delta}_\omega^+(z) \Theta_\varphi^\omega n^c(x_1)] [\bar{p}^c(x_2) \gamma^\mu n(x_2) - \bar{n}^c(x_2) \gamma^\mu p(x_2)] [\bar{p}(x_3) \gamma_\beta \gamma^5 \Delta_\alpha^+(x_3)]) | 0 > \right. \\
&- < 0 | T([\bar{\Delta}_\omega^0(z) \Theta_\varphi^\omega p^c(x)] [\bar{p}^c(x_2) \gamma^\mu n(x_2) - \bar{n}^c(x_2) \gamma^\mu p(x_2)] \\
&\times [\bar{n}(x_3) \gamma_\beta \gamma^5 \Delta_\alpha^0(x_3)]) | 0 > + (\gamma^5 \otimes 1 \rightarrow -\gamma_\nu \gamma^5 \otimes \gamma^\nu) \left. \right\} = \\
&= \frac{i}{3} \frac{eg_V}{M_N^2} \frac{g_{\pi N\Delta}}{g_{\pi NN}} \frac{g_{\gamma N\Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} \int d^4x_1 d^4x_2 d^4x_3 \int d^4z \frac{\partial}{\partial z_\varphi} \delta^{(4)}(z - x_1) \\
&\times T([p^c(x_1) \gamma^5 n(z) + \bar{p}^c(z) \gamma^5 n(x_1)] D_\mu^\dagger(x_2) F^{\alpha\beta}(x_3)) \\
&\times \left\{ < 0 | T([\bar{\Delta}_\omega^+(z) \Theta_\varphi^\omega n^c(x_1)] [\bar{n}^c(x_2) \gamma^\mu p(x_2)] [\bar{p}(x_3) \gamma_\beta \gamma^5 \Delta_\alpha^+(x_3)]) | 0 > \right. \\
&+ < 0 | T([\bar{\Delta}_\omega^0(z) \Theta_\varphi^\omega p^c(x_1)] [\bar{p}^c(x_2) \gamma^\mu n(x_2)] [\bar{n}(x_3) \gamma_\beta \gamma^5 \Delta_\alpha^0(x_3)]) | 0 > \\
&+ (\gamma^5 \otimes 1 \rightarrow -\gamma_\nu \gamma^5 \otimes \gamma^\nu) \left. \right\} = \\
&= \frac{2}{3} \frac{ieg_V}{M_N^2} \frac{g_{\pi N\Delta}}{g_{\pi NN}} \frac{g_{\gamma N\Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} \int d^4x_1 d^4x_2 d^4x_3 \int d^4z \frac{\partial}{\partial z_\varphi} \delta^{(4)}(z - x_1) \\
&\times \left\{ T([p^c(x_1) \gamma^5 n(z) + \bar{p}^c(z) \gamma^5 n(x_1)] D_\mu^\dagger(x_2) F^{\alpha\beta}(x_3)) \right. \\
&\times \frac{1}{i} \text{tr}\{S_{\alpha\omega}(x_3 - z) \Theta_\varphi^\omega S_F^c(x_1 - x_2) \gamma^\mu S_F(x_2 - x_3) \gamma_\beta \gamma^5\} \\
&- T([p^c(x_1) \gamma_\nu \gamma^5 n(z) + \bar{p}^c(z) \gamma_\nu \gamma^5 n(x_1)] D_\mu^\dagger(x_2) F^{\alpha\beta}(x_3)) \\
&\times \frac{1}{i} \text{tr}\{S_{\alpha\omega}(x_3 - z) \Theta_\varphi^\omega \gamma^\nu S_F^c(x_1 - x_2) \gamma^\mu S_F(x_2 - x_3) \gamma_\beta \gamma^5\} \left. \right\}. \tag{3.9}
\end{aligned}$$

Thus, the effective Lagrangian $\mathcal{L}_{np \rightarrow \Delta N \rightarrow D\gamma}(x)$ reads

$$\begin{aligned}
\int d^4x \mathcal{L}_{np \rightarrow \Delta N \rightarrow D\gamma}(x) &= \frac{2}{3} \frac{ieg_V}{M_N^2} \frac{g_{\pi N\Delta}}{g_{\pi NN}} \frac{g_{\gamma N\Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} \int d^4x_1 d^4x_2 d^4x_3 \int d^4z \frac{\partial}{\partial z_\varphi} \delta^{(4)}(z - x_1) \\
&\times \left\{ T([p^c(x_1) \gamma^5 n(z) + \bar{p}^c(z) \gamma^5 n(x_1)] D_\mu^\dagger(x_2) F^{\alpha\beta}(x_3)) \right. \\
&\times \frac{1}{i} \text{tr}\{S_{\alpha\omega}(x_3 - z) \Theta_\varphi^\omega S_F^c(x_1 - x_2) \gamma^\mu S_F(x_2 - x_3) \gamma_\beta \gamma^5\} \\
&- T([p^c(x_1) \gamma_\nu \gamma^5 n(z) + \bar{p}^c(z) \gamma_\nu \gamma^5 n(x_1)] D_\mu^\dagger(x_2) F^{\alpha\beta}(x_3)) \\
&\times \frac{1}{i} \text{tr}\{S_{\alpha\omega}(x_3 - z) \Theta_\varphi^\omega \gamma^\nu S_F^c(x_1 - x_2) \gamma^\mu S_F(x_2 - x_3) \gamma_\beta \gamma^5\} \left. \right\}. \tag{3.10}
\end{aligned}$$

In the momentum representation of the baryon Green functions the effective Lagrangian Eq.(3.10) reads

$$\int d^4x \mathcal{L}_{np \rightarrow \Delta N \rightarrow D\gamma}(x) = \frac{2}{3} \frac{ieg_V}{M_N^2} \frac{g_{\pi N\Delta}}{g_{\pi NN}} \frac{g_{\gamma N\Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} \int d^4x_1 \int d^4z \frac{\partial}{\partial z_\varphi} \delta^{(4)}(z - x_1)$$

$$\begin{aligned}
& \times \int \frac{d^4 x_2 d^4 k_2}{(2\pi)^4} \frac{d^4 x_3 d^4 k_3}{(2\pi)^4} e^{-ik_2 \cdot (x_2 - x_1)} e^{-ik_3 \cdot (x_3 - z)} \\
& \times \left\{ T([p^c(x_1)\gamma^5 n(z) + \bar{p}^c(z)\gamma^5 n(x_1)] D_\mu^\dagger(x_2) F_{\alpha\beta}(x_3)) \right. \\
& \times \int \frac{d^4 k_1}{\pi^2 i} e^{ik_1 \cdot (x_1 - z)} \text{tr}\{S^{\alpha\omega}(k_1 + k_3) \Theta_{\omega\varphi} \frac{1}{M_N - \hat{k}_1 + \hat{k}_2} \gamma^\mu \frac{1}{M_N - \hat{k}_1} \gamma^\beta \gamma^5\} \\
& - T([p^c(x_1)\gamma_\nu \gamma^5 n(z) + \bar{p}^c(z)\gamma_\nu \gamma^5 n(x_1)] D_\mu^\dagger(x_2) F^{\alpha\beta}(x_3)) \\
& \times \left. \int \frac{d^4 k_1}{\pi^2 i} e^{ik_1 \cdot (x_1 - z)} \text{tr}\{S^{\alpha\omega}(k_1 + k_3) \Theta_{\omega\varphi} \frac{1}{M_N - \hat{k}_1 + \hat{k}_2} \gamma^\mu \frac{1}{M_N - \hat{k}_1} \gamma^\beta \gamma^5\} \right\}. \quad (3.11)
\end{aligned}$$

The effective Lagrangian Eq.(3.11) defines the contribution of the Δ resonance to the amplitude of the neutron–proton radiative capture.

The amplitude of the neutron–proton radiative capture caused by the contribution of the $\Delta(1232)$ resonance exchange we define by a usual way [5]:

$$\begin{aligned}
& \int d^4 x \langle D(k_D) \gamma(k) | \mathcal{L}_{np \rightarrow \Delta N \rightarrow D \gamma}(x) | n(p_1) p(p_2) \rangle = \\
& = (2\pi)^4 \delta^{(4)}(k_D + k - p_1 - p_2) \frac{\mathcal{M}(n + p \rightarrow \Delta N \rightarrow D + \gamma)}{\sqrt{2E_1 V 2E_2 V 2E_D V 2\omega V}}, \quad (3.12)
\end{aligned}$$

where E_i ($i = 1, 2, D$) and ω are the energies of the neutron, the proton, the deuteron and the photon, and V is the normalization volume. For the computation of the amplitude $\mathcal{M}(n + p \rightarrow \Delta N \rightarrow D + \gamma)$ we should take the r.h.s. of Eq.(3.11) between the wave functions of the initial $|n(p_1) p(p_2)\rangle$ and the final $\langle D(k_D) \gamma(k)|$ states. This gives

$$\begin{aligned}
& (2\pi)^4 \delta^{(4)}(k_D + k - p_1 - p_2) \frac{\mathcal{M}(n + p \rightarrow \Delta N \rightarrow D + \gamma)}{\sqrt{2E_1 V 2E_2 V 2E_D V 2\omega V}} = \\
& = \frac{2ieg_V}{3} \frac{g_{\pi N \Delta}}{M_N^2} \frac{g_{\gamma N \Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} \int d^4 x_1 \int d^4 z \frac{\partial}{\partial z_\varphi} \delta^{(4)}(z - x_1) \\
& \times \int \frac{d^4 x_2 d^4 k_2}{(2\pi)^4} \frac{d^4 x_3 d^4 k_3}{(2\pi)^4} e^{-ik_2 \cdot (x_2 - x_1)} e^{-ik_3 \cdot (x_3 - z)} \\
& \times \left\{ \langle D(k_D) \gamma(k) | T([p^c(x_1)\gamma^5 n(z) + \bar{p}^c(z)\gamma^5 n(x_1)] D_\mu^\dagger(x_2) F_{\alpha\beta}(x_3)) | n(p_1) p(p_2) \rangle \right. \\
& \times \int \frac{d^4 k_1}{\pi^2 i} e^{ik_1 \cdot (x_1 - z)} \text{tr}\{S^{\alpha\omega}(k_1 + k_3) \Theta_{\omega\varphi} \frac{1}{M_N - \hat{k}_1 + \hat{k}_2} \gamma^\mu \frac{1}{M_N - \hat{k}_1} \gamma^\beta \gamma^5\} \\
& - \langle D(k_D) \gamma(k) | T([p^c(x_1)\gamma_\nu \gamma^5 n(z) + \bar{p}^c(z)\gamma_\nu \gamma^5 n(x_1)] D_\mu^\dagger(x_2) F_{\alpha\beta}(x_3)) | n(p_1) p(p_2) \rangle \\
& \times \left. \int \frac{d^4 k_1}{\pi^2 i} e^{ik_1 \cdot (x_1 - z)} \text{tr}\{S^{\alpha\omega}(k_1 + k_3) \Theta_{\omega\varphi} \frac{1}{M_N - \hat{k}_1 + \hat{k}_2} \gamma^\mu \frac{1}{M_N - \hat{k}_1} \gamma^\beta \gamma^5\} \right\}. \quad (3.13)
\end{aligned}$$

The matrix elements between the initial and the final states are given by [5]:

$$\begin{aligned}
& \langle D(k_D) \gamma(k) | T([p^c(x_1)\gamma^5 n(z) + \bar{p}^c(z)\gamma^5 n(x_1)] D_\mu^\dagger(x_2) F_{\alpha\beta}(x_3)) | n(p_1) p(p_2) \rangle = \\
& = [\bar{u}^c(p_2) \gamma^5 u(p_1)] \times i(k_\alpha e_\beta^*(k) - k_\beta e_\alpha^*(k) \times e_\mu^*(k_D)) \\
& \times \frac{e^{ik_D \cdot x_2} e^{ik \cdot x_3}}{\sqrt{2E_1 V 2E_2 V 2E_D V 2\omega V}} \left(e^{-ip_1 \cdot x_1 - ip_2 \cdot z} + e^{-ip_2 \cdot x_1 - ip_1 \cdot z} \right),
\end{aligned}$$

$$\begin{aligned}
& \langle D(k_D) \gamma(k) | T([\bar{p}^c(x_1) \gamma_\nu \gamma^5 n(z) + \bar{p}^c(z) \gamma_\nu \gamma^5 n(x_1)] D_\mu^\dagger(x_2) F_{\alpha\beta}(x_3)) | n(p_1) p(p_2) \rangle = \\
& = [\bar{u}^c(p_2) \gamma_\nu \gamma^5 u(p_1)] \times i (k_\alpha e_\beta^*(k) - k_\beta e_\alpha^*(k)) \times e_\mu^*(k_D) \\
& \times \frac{e^{ik_D \cdot x_2} e^{ik \cdot x_3}}{\sqrt{2E_1 V 2E_2 V 2E_D V 2\omega V}} \left(e^{-ip_1 \cdot x_1 - ip_2 \cdot z} + e^{-ip_2 \cdot x_1 - ip_1 \cdot z} \right). \quad (3.14)
\end{aligned}$$

Substituting Eq.(3.14) in Eq.(3.13) and integrating over z we obtain

$$\begin{aligned}
& (2\pi)^4 \delta^{(4)}(k_D + k - p_1 - p_2) \mathcal{M}(n + p \rightarrow \Delta N \rightarrow D + \gamma) = \\
& = -\frac{ie}{2M_N^2} \frac{g_V}{6\pi^2} \frac{g_{\pi N \Delta}}{g_{\pi NN}} \frac{g_{\gamma N \Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} [\bar{u}^c(p_2) \gamma^5 u(p_1)] (k_\alpha e_\beta^*(k) - k_\beta e_\alpha^*(k)) e_\mu^*(k_D) \\
& \times \int d^4 x_1 \int \frac{d^4 x_2 d^4 k_2}{(2\pi)^4} \frac{d^4 x_3 d^4 k_3}{(2\pi)^4} e^{i(k_2 + k_3 - p_1 - p_2) \cdot x_1} e^{i(k_D - k_2) \cdot x_2} e^{i(k - k_3) \cdot x_3} \\
& \times \int \frac{d^4 k_1}{\pi^2 i} \text{tr}\{(p_1 + p_2 + k_1 - k_3)^\varphi S^{\alpha\omega}(k_1 + k_3) \Theta_{\omega\varphi} \frac{1}{M_N - \hat{k}_1 + \hat{k}_2} \gamma^\mu \frac{1}{M_N - \hat{k}_1} \gamma_\beta \gamma^5\} \\
& + \frac{ie}{2M_N^2} \frac{g_V}{6\pi^2} \frac{g_{\pi N \Delta}}{g_{\pi NN}} \frac{g_{\gamma N \Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} [\bar{u}^c(p_2) \gamma_\nu \gamma^5 u(p_1)] (k_\alpha e_\beta^*(k) - k_\beta e_\alpha^*(k)) e_\mu^*(k_D) \\
& \times \int d^4 x_1 \int \frac{d^4 x_2 d^4 k_2}{(2\pi)^4} \frac{d^4 x_3 d^4 k_3}{(2\pi)^4} e^{i(k_2 + k_3 - p_1 - p_2) \cdot x_1} e^{i(k_D - k_2) \cdot x_2} e^{i(k - k_3) \cdot x_3} \\
& \times \int \frac{d^4 k_1}{\pi^2 i} \text{tr}\{(p_1 + p_2 + k_1 - k_3)^\varphi S^{\alpha\omega}(k_1 + k_3) \Theta_{\omega\varphi} \gamma^\nu \frac{1}{M_N - \hat{k}_1 + \hat{k}_2} \gamma^\mu \frac{1}{M_N - \hat{k}_1} \gamma_\beta \gamma^5\}. \quad (3.15)
\end{aligned}$$

Integrating over x_1, x_2, x_3, k_2 and k_3 we obtain in the r.h.s. of Eq.(3.15) the δ -function describing the 4-momentum conservation. The, the amplitude $\mathcal{M}(n + p \rightarrow \Delta N \rightarrow D + \gamma)$ becomes equal

$$\begin{aligned}
& \mathcal{M}(n + p \rightarrow \Delta N \rightarrow D + \gamma) = \\
& = -\frac{ie}{2M_N^2} \frac{g_V}{6\pi^2} \frac{g_{\pi N \Delta}}{g_{\pi NN}} \frac{g_{\gamma N \Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} [\bar{u}^c(p_2) \gamma^5 u(p_1)] (k_\alpha e_\beta^*(k) - k_\beta e_\alpha^*(k)) e_\mu^*(k_D) \\
& \times \int \frac{d^4 k_1}{\pi^2 i} \text{tr}\{(k_1 + k_D)^\varphi S^{\alpha\omega}(k_1 + k) \Theta_{\omega\varphi} \frac{1}{M_N - \hat{k}_1 + \hat{k}_D} \gamma^\mu \frac{1}{M_N - \hat{k}_1} \gamma_\beta \gamma^5\} \\
& + \frac{e}{2M_N^2} \frac{g_V}{6\pi^2} \frac{g_{\pi N \Delta}}{g_{\pi NN}} \frac{g_{\gamma N \Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} [\bar{u}^c(p_2) \gamma_\nu \gamma^5 u(p_1)] (k_\alpha e_\beta^*(k) - k_\beta e_\alpha^*(k)) e_\mu^*(k_D) \\
& \times \int \frac{d^4 k_1}{\pi^2 i} \text{tr}\{(k_1 + k_D)^\varphi S^{\alpha\omega}(k_1 + k) \Theta_{\omega\varphi} \gamma^\nu \frac{1}{M_N - \hat{k}_1 + \hat{k}_D} \gamma^\mu \frac{1}{M_N - \hat{k}_1} \gamma_\beta \gamma^5\}, \quad (3.16)
\end{aligned}$$

For the subsequent calculation it is convenient to introduce the structure functions

$$\begin{aligned}
& \mathcal{M}(n + p \rightarrow \Delta N \rightarrow D + \gamma) = \\
& = -\frac{ie}{2M_N^2} \frac{g_V}{6\pi^2} \frac{g_{\pi N \Delta}}{g_{\pi NN}} \frac{g_{\gamma N \Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} [\bar{u}^c(p_2) \gamma^5 u(p_1)] (k_\alpha e_\beta^*(k) - k_\beta e_\alpha^*(k)) e_\mu^*(k_D) \\
& \times \mathcal{J}_5^{\mu\beta\alpha}(k_D, k) \\
& + \frac{ie}{2M_N^2} \frac{g_V}{6\pi^2} \frac{g_{\pi N \Delta}}{g_{\pi NN}} \frac{g_{\gamma N \Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} [\bar{u}^c(p_2) \gamma_\nu \gamma^5 u(p_1)] (k_\alpha e_\beta^*(k) - k_\beta e_\alpha^*(k)) e_\mu^*(k_D) \\
& \times \mathcal{J}_5^{\nu\mu\beta\alpha}(k_D, k), \quad (3.17)
\end{aligned}$$

where the structure functions $\mathcal{J}_5^{\mu\beta\alpha}(k_D, k)$ and $\mathcal{J}_5^{\nu\mu\beta\alpha}(k_D, k)$ are defined by the momentum integrals

$$\begin{aligned}\mathcal{J}_5^{\mu\beta\alpha}(k_D, k) &= \\ &= \int \frac{d^4 k_1}{\pi^2 i} \text{tr}\{(k_1 + k_D)^\varphi S^{\alpha\omega}(k_1 + k) \Theta_{\omega\varphi} \frac{1}{M_N - \hat{k}_1 + \hat{k}_D} \gamma^\mu \frac{1}{M_N - \hat{k}_1} \gamma^\beta \gamma^5\}, \\ \mathcal{J}_5^{\nu\mu\beta\alpha}(k_D, k) &= \\ &= \int \frac{d^4 k_1}{\pi^2 i} \text{tr}\{(k_1 + k_D)^\varphi S^{\alpha\omega}(k_1 + k) \Theta_{\omega\varphi} \gamma^\nu \frac{1}{M_N - \hat{k}_1 + \hat{k}_D} \gamma^\mu \frac{1}{M_N - \hat{k}_1} \gamma^\beta \gamma^5\}. \quad (3.18)\end{aligned}$$

Now the problem of the calculation of the contribution of the $\Delta(1232)$ resonance to the amplitude of the neutron–proton radiative capture is reduced to the problem of the evaluation of the structure functions. In the leading order in large N_C expansion [1] we obtain

$$\begin{aligned}\mathcal{J}_5^{\mu\beta\alpha}(k_D, k) &= \frac{4}{3} \left(Z - \frac{1}{2} \right) i M_N \varepsilon^{\mu\beta\alpha\lambda} k_{D\lambda}, \\ \mathcal{J}_5^{\nu\mu\beta\alpha}(k_D, k) &= \frac{2}{3} \left(Z - \frac{1}{2} \right) i M_N^2 \varepsilon^{\mu\beta\alpha\nu}. \quad (3.19)\end{aligned}$$

We have neglected the mass difference between the masses of the $\Delta(1232)$ resonance and the nucleon. The amplitude of the neutron–proton radiative capture caused by the $\Delta(1232)$ resonance contribution is equal to

$$\begin{aligned}\mathcal{M}_{\Delta}(n + p \rightarrow D + \gamma) &= \frac{e}{2M_N} \frac{g_V}{4\pi^2} \left[\left(\frac{1}{2} - Z \right) \frac{8}{9} \frac{g_{\pi N\Delta}}{g_{\pi NN}} \frac{g_{\gamma N\Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} \right] \\ &\times \varepsilon^{\alpha\beta\mu\nu} k_\alpha e_\beta^*(k) e_\mu^*(k_D) [\bar{u}^c(p_2) (2k_{D\nu} - M_N \gamma_\nu) \gamma^5 u(p_1)] = \\ &= e \frac{5g_V}{8\pi^2} \left[\left(\frac{1}{2} - Z \right) \frac{8}{9} \frac{g_{\pi N\Delta}}{g_{\pi NN}} \frac{g_{\gamma N\Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} \right] (\vec{k} \times \vec{e}^*(\vec{k})) \cdot \vec{e}^*(\vec{k}_D) [\bar{u}^c(p_2) \gamma^5 u(p_1)]. \quad (3.20)\end{aligned}$$

The total amplitude of the neutron–proton radiative capture for thermal neutrons reads

$$\begin{aligned}\mathcal{M}(n + p \rightarrow D + \gamma) &= e (\mu_p - \mu_n) \frac{5g_V}{8\pi^2} G_{\pi NN} (\vec{k} \times \vec{e}^*(\vec{k})) \cdot \vec{e}^*(\vec{k}_D) [\bar{u}^c(p_2) \gamma^5 u(p_1)] \\ &\times \left[1 + \frac{g_{\pi NN}^2}{\mu_p - \mu_n} \frac{M_\pi^2}{8\pi^2} \frac{\alpha_\rho}{\pi} \left(J_{\pi a_1 N} + \frac{3}{2g_A} J_{\pi V N} \right) + \frac{1 - 2Z}{\mu_p - \mu_n} \frac{1}{G_{\pi NN}} \frac{4}{9} \frac{g_{\pi N\Delta}}{g_{\pi NN}} \frac{g_{\gamma N\Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} \right]. \quad (3.21)\end{aligned}$$

The total cross section for the neutron–proton radiative capture is then defined by

$$\begin{aligned}\sigma^{\text{np}}(k) &= \frac{1}{v} (\mu_p - \mu_n)^2 \frac{25}{64} \frac{\alpha}{\pi^2} Q_D G_{\pi NN}^2 M_N \varepsilon_D^3 \\ &\times \left[1 + \frac{g_{\pi NN}^2}{\mu_p - \mu_n} \frac{M_\pi^2}{8\pi^2} \frac{\alpha_\rho}{\pi} \left(J_{\pi a_1 N} + \frac{3}{2g_A} J_{\pi V N} \right) + \frac{1 - 2Z}{\mu_p - \mu_n} \frac{1}{G_{\pi NN}} \frac{4}{9} \frac{g_{\pi N\Delta}}{g_{\pi NN}} \frac{g_{\gamma N\Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} \right]^2. \quad (3.22)\end{aligned}$$

The numerical value of the cross section amounts to

$$\sigma^{\text{np}}(k) = 287.2 (1 + 0.64 (1 - 2Z))^2 \text{ m b}. \quad (3.23)$$

Thus, the discrepancy of the theoretical cross section and the experimental value $\sigma_{\text{exp}}^{\text{np}} = (334.2 \pm 0.5) \text{ mb}$ can be described by the contribution of the $\Delta(1232)$ resonance. In order to fit the experimental value of the cross section we should take Z equal to $Z = 0.438$. This agrees with the experimental bound $|Z| \leq 1/2$ [23].

4 Photo–magnetic disintegration of the deuteron

The amplitude of the photo–magnetic disintegration of the deuteron $\gamma + D \rightarrow n + p$ evaluated near threshold is related to the amplitude of the neutron–proton radiative capture $n + p \rightarrow D + \gamma$ and reads

$$\begin{aligned} \mathcal{M}(\gamma + D \rightarrow n + p) &= e(\mu_p - \mu_n) \frac{5g_V}{8\pi^2} G_{\pi NN} (\vec{q} \times \vec{e}(\vec{q})) \cdot \vec{e}(\vec{k}_D) [\bar{u}(p_2) \gamma^5 u^c(p_1)] \\ &\times \left[1 + \frac{g_{\pi NN}^2}{\mu_p - \mu_n} \frac{M_\pi^2}{8\pi^2} \frac{\alpha_\rho}{\pi} \left(J_{\pi a_1 N} + \frac{3}{2g_A} J_{\pi V N} \right) + \frac{1-2Z}{\mu_p - \mu_n} \frac{1}{G_{\pi NN}} \frac{4}{9} \frac{g_{\pi N \Delta}}{g_{\pi NN}} \frac{g_{\gamma N \Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} \right]. \end{aligned} \quad (4.1)$$

The cross section defined by the amplitude Eq.(4.1) is then given by

$$\sigma^{\gamma D}(\omega) = \sigma_0 \left(\frac{\omega}{\varepsilon_D} \right) k r_D, \quad (4.2)$$

where $k = \sqrt{M_N(\omega - \varepsilon_D)}$ is the relative momentum of the np system, ω is the energy of the photon and $r_D = 1/\sqrt{\varepsilon_D M_N} = 4.315$ fm is the radius of the deuteron, and σ_0 is equal to

$$\begin{aligned} \sigma_0 &= (\mu_p - \mu_n)^2 \frac{25\alpha Q_D}{192\pi^2} G_{\pi NN}^2 \varepsilon_D^{3/2} M_N^{5/2} \\ &\times \left[1 + \frac{g_{\pi NN}^2}{\mu_p - \mu_n} \frac{M_\pi^2}{8\pi^2} \frac{\alpha_\rho}{\pi} \left(J_{\pi a_1 N} + \frac{3}{2g_A} J_{\pi V N} \right) + \frac{1-2Z}{\mu_p - \mu_n} \frac{1}{G_{\pi NN}} \frac{4}{9} \frac{g_{\pi N \Delta}}{g_{\pi NN}} \frac{g_{\gamma N \Delta}}{g_{\pi NN}} \frac{g_{\pi NN}^3}{4M_\pi^2} \right]^2 = \\ &= 7.10 \text{ m b}. \end{aligned} \quad (4.3)$$

The cross section $\sigma^{\gamma D}(\omega)$ calculated in the PMA near threshold has the same form as Eq. (4.2) but with σ_0 amounting to [17]

$$\sigma_0 = \frac{2\pi\alpha}{3M_N^2} (\mu_p - \mu_n)^2 \left(1 - a_{np} \sqrt{\varepsilon_D M_N} \right)^2 = 6.31 \text{ mb}. \quad (4.4)$$

It is seen that numerical values of σ_0 defined by Eq. (4.3) and Eq. (4.4) are in good agreement. We should emphasize that the cross section Eq.(4.4) does not contain corrections mentioned by Riska and Brown [11] (see also [17]) increasing its value.

In order to obtain the cross section for the process $\gamma + D \rightarrow n + p$ far from threshold we take into account the np interaction in the final state. This can be carried out by summing up an infinite series of one–nucleon loop diagrams. In this case the amplitude of $\gamma + D \rightarrow n + p$ reads

$$\begin{aligned} \mathcal{M}(\gamma + D \rightarrow n + p) &= \mathcal{A}_{\text{th}} [\bar{u}(p_2) \gamma^5 u^c(p_1)] \\ &\times \frac{1}{1 + \frac{G_{\pi NN}}{16\pi^2} \int \frac{d^4 p}{\pi^2 i} \text{tr} \left\{ \gamma^5 \frac{1}{M_N - \hat{p} - \hat{P} - \hat{Q}} \gamma^5 \frac{1}{M_N - \hat{p} - \hat{Q}} \right\}}, \end{aligned} \quad (4.5)$$

where \mathcal{A}_{th} is the amplitude calculated near threshold, $P = p_1 + p_2 = (2\sqrt{k^2 + M_N^2}, \vec{0})$ is the 4–momentum of the np system in the center of mass frame. Then, $Q = aP + bK = a(p_1 + p_2) + b(p_1 - p_2)$ is an arbitrary shift of virtual momentum with arbitrary parameters

a and b , and in the center of mass frame $K = p_1 - p_2 = (0, 2\vec{k})$. The explicit dependence of the momentum integral on Q can be evaluated by means of the Gertsein–Jackiw procedure [28] and is given by [1–6]:

$$\begin{aligned} & \int \frac{d^4 p}{\pi^2 i} \text{tr} \left\{ \gamma^5 \frac{1}{M_N - \hat{p} - \hat{P} - \hat{Q}} \gamma^5 \frac{1}{M_N - \hat{p} - \hat{Q}} \right\} = \\ & = \int \frac{d^4 p}{\pi^2 i} \text{tr} \left\{ \gamma^5 \frac{1}{M_N - \hat{p} - \hat{P}} \gamma^5 \frac{1}{M_N - \hat{p}} \right\} - 2a(a+1)P^2 - 2b^2 K^2. \end{aligned} \quad (4.6)$$

For the evaluation of the momentum integral over p we would keep only the leading order contributions in the large N_C expansion [1]. This yields

$$\begin{aligned} & \int \frac{d^4 p}{\pi^2 i} \text{tr} \left\{ \gamma^5 \frac{1}{M_N - \hat{p} - \hat{P} - \hat{Q}} \gamma^5 \frac{1}{M_N - \hat{p} - \hat{Q}} \right\} = \\ & = -8a(a+1)M_N^2 + 8(b^2 - a(a+1))k^2 - i8\pi M_N k. \end{aligned} \quad (4.7)$$

The amplitude Eq.(4.5) we obtain in the form

$$\mathcal{M}(\gamma + D \rightarrow n + p) = \mathcal{A}_{\text{th}} [\bar{u}(p_2) \gamma^5 u^c(p_1)] \frac{Z}{1 - \frac{1}{2} r_{\text{np}} a_{\text{np}} k^2 + i a_{\text{np}} k}. \quad (4.8)$$

Here we have denoted

$$\begin{aligned} a_{\text{np}} &= -\frac{G_{\pi\text{NN}} M_N}{2\pi} Z, \quad r_{\text{np}} = (b^2 - a(a+1)) \frac{2}{\pi} \frac{1}{M_N}, \\ \frac{1}{Z} &= 1 - \frac{a(a+1)}{2\pi^2} G_{\pi\text{NN}} M_N^2, \end{aligned} \quad (4.9)$$

where $r_{\text{np}} = 2.75 \pm 0.05$ fm is the effective range of low-energy elastic np scattering.

Renormalizing the wave functions of nucleons $\sqrt{Z} u(p_1) \rightarrow u(p_1)$ and $\sqrt{Z} u(p_2) \rightarrow u(p_2)$ we arrive at the amplitude of the photo-magnetic disintegration of the deuteron

$$\mathcal{M}(\gamma + D \rightarrow n + p) = \frac{\mathcal{A}_{\text{th}}}{1 - \frac{1}{2} r_{\text{np}} a_{\text{np}} k^2 + i a_{\text{np}} k} [\bar{u}(p_2) \gamma^5 u^c(p_1)], \quad (4.10)$$

where the factor $1/(1 - \frac{1}{2} r_{\text{np}} a_{\text{np}} k^2 + i a_{\text{np}} k)$ describes the contribution of low-energy elastic np scattering in agreement with low-energy nuclear phenomenology [17]. The amplitude Eq.(4.10) yields the cross section

$$\sigma^{\gamma D}(\omega) = \sigma_0 \left(\frac{\omega}{\varepsilon_D} \right) \frac{k r_D}{\left(1 - \frac{1}{2} r_{\text{np}} a_{\text{np}} k^2 \right)^2 + a_{\text{np}}^2 k^2}. \quad (4.11)$$

At zero effective range $r_{\text{np}} = 0$ the cross section Eq.(4.11) reduces to the form

$$\sigma^{\gamma D}(\omega) = \sigma_0 \left(\frac{\omega}{\varepsilon_D} \right) \frac{k r_D}{1 + a_{\text{np}}^2 k^2}. \quad (4.12)$$

In the PMA [17], in turn, at zero effective range the cross section $\sigma^{\gamma D}(\omega)$ has been found in the form

$$\sigma^{\gamma D}(\omega) = \sigma_0 \left(\frac{\omega}{\varepsilon_D} \right) \frac{kr_D}{1 + a_{np}^2 k^2} \frac{1}{(1 + r_D^2 k^2)^2}. \quad (4.13)$$

It is seen that Eqs.(4.13) and (4.12) differ by a factor $1/(1 + r_D^2 k^2)^2$. This factor as well as the dependence on the S-wave scattering length and the effective range [17] appears by virtue of the integral of the overlap of the wave functions of the deuteron $\psi_D(r)$ and the relative movement of the np-system in the 1S_0 -state $\psi_{np}(kr)$.

In order to introduce in the RFMD the wave functions of the deuteron and the relative movement of the np-pair we can follow Bohr and Mottelson [29] and: (1) in the initial nuclear state $|D(k_D)\gamma(q) \rangle = a_D^\dagger(\vec{k}_D, \lambda_D) a^\dagger(\vec{q}, \lambda)|0 \rangle$ represent the operator of creation of the deuteron $a_D^\dagger(\vec{k}_D, \lambda_D)$ with 3-momentum \vec{k}_D and polarization λ_D in terms of the operators of creation of the proton $a_p^\dagger(\vec{p}, \sigma_p)$ and the neutron $a_n^\dagger(\vec{k}_D - \vec{p}, \sigma_n)$ and (2) in the final nuclear state $\langle n(p_2)p(p_1) | = \langle 0 | a_n(\vec{p}_2, \sigma_2) a_p(\vec{p}_1, \sigma_1)$ replace the product of the operators of annihilation of the neutron and the proton by the operator of annihilation of the np-pair in the 1S_0 -state $a_n(\vec{p}_2, \sigma_2) a_p(\vec{p}_1, \sigma_1) \rightarrow a_{np}(\vec{P}, \vec{k}; S=0)$, where $\vec{P} = \vec{p}_1 + \vec{p}_2$, $\vec{k} = (\vec{p}_1 - \vec{p}_2)/2$ and $S=0$ is a total spin. In the form adjusted to our problem these changes read

$$\begin{aligned} a_D^\dagger(\vec{k}_D, \lambda_D) &\sim \sum_{\sigma_p, \sigma_n = \pm 1} \int \frac{d^3 p}{\sqrt{2E_{\vec{p}} 2E_{\vec{k}_D - \vec{p}}}} e_\mu(\vec{k}_D, \lambda_D) [\bar{u}(\vec{p}, \sigma_p) \gamma^\mu u^c(\vec{k}_D - \vec{p}, \sigma_n)] \\ &\times a_p^\dagger(\vec{p}, \sigma_p) a_n^\dagger(\vec{k}_D - \vec{p}, \sigma_n) \int d^3 r \psi_D(r) e^{i(\vec{p} - \vec{k}_D/2) \cdot \vec{r}}, \\ a_{np}(\vec{P}, \vec{k}; S=0) &\sim \sum_{\sigma_1, \sigma_2 = \pm 1} \int \frac{d^3 p}{\sqrt{2E_{\vec{p}} 2E_{\vec{P} - \vec{p}}}} [\bar{u}^c(\vec{p}, \sigma_2) \gamma^5 u(\vec{P} - \vec{p}, \sigma_1)] \\ &\times a_n(\vec{p}, \sigma_2) a_p(\vec{P} - \vec{p}, \sigma_1) \int d^3 r \psi_{np}^*(kr) e^{-i(\vec{p} - \vec{P}/2) \cdot \vec{r}}, \end{aligned} \quad (4.14)$$

where $E_{\vec{p}} = \sqrt{\vec{p}^2 + M_N^2}$. The spinorial parts of the wave functions of the deuteron and the np-pair are given in terms of the Dirac bispinors in the relativistically covariant form. The operators Eq.(4.14) can be involved into evaluation of low-energy nuclear matrix elements through the reduction technique [30].

However, such a modification complicates the model substantially and goes beyond the scope of this paper. Therefore, referring to the possibility to describing the factor $1/(1 + r_D^2 k^2)^2$ correctly in the RFMD the problem of this factor can be preferably solved phenomenologically. In fact, since this factor is universal for all processes of the deuteron coupled to the NN system in the 1S_0 -state at low energies, we suggest to multiply by a factor $1/(1 + r_D^2 k^2)$ any amplitude of low-energy nuclear process of this kind obtained near threshold, i.e. defined by the corresponding effective Lagrangian evaluated through one-nucleon loop exchanges in leading order in the large N_C expansion [1]. This implies the change

$$\mathcal{A}_{th} \rightarrow \frac{\mathcal{A}_{th}}{1 + r_D^2 k^2}. \quad (4.15)$$

In other words, we introduce an universal form factor

$$F_D(k^2) = \frac{1}{1 + r_D^2 k^2} \quad (4.16)$$

describing a spatial smearing of the deuteron coupled to the NN system in the 1S_0 -state at low energies.

As a result the amplitude of the photo-magnetic disintegration of the deuteron obtained in the RFMD is equal to

$$\mathcal{M}(\gamma + D \rightarrow n + p) = \frac{\mathcal{A}_{\text{th}}}{1 - \frac{1}{2} r_{\text{np}} a_{\text{np}} k^2 + i a_{\text{np}} k} F_D(k^2) [\bar{u}(p_2) \gamma^5 u^c(p_1)]. \quad (4.17)$$

The cross section for the photo-magnetic disintegration of the deuteron evaluated in the RFMD reads

$$\begin{aligned} \sigma^{\gamma D}(\omega) &= \sigma_0 \left(\frac{\omega}{\varepsilon_D} \right) \frac{k r_D}{\left(1 - \frac{1}{2} r_{\text{np}} a_{\text{np}} k^2 \right)^2 + a_{\text{np}}^2 k^2} F_D^2(k^2) = \\ &= \sigma_0 \frac{k r_D}{1 + r_D^2 k^2} \frac{1}{\left(1 - \frac{1}{2} r_{\text{np}} a_{\text{np}} k^2 \right)^2 + a_{\text{np}}^2 k^2}, \end{aligned} \quad (4.18)$$

where σ_0 is given by Eq.(4.2).

5 Anti-neutrino disintegration of the deuteron via charged weak current interaction

The effective Lagrangian describing the low-energy nuclear transition $\bar{\nu}_e + D \rightarrow e^+ + n + n$ has been calculated in [5,6] through one-nucleon loop exchanges and in leading order in the large N_C expansion [1]:

$$\mathcal{L}_{\bar{\nu}_e D \rightarrow e^+ nn}(x) = -i g_A M_N G_{\pi NN} \frac{G_V}{\sqrt{2}} \frac{3g_V}{4\pi^2} D_\mu(x) [\bar{n}(x) \gamma^5 n^c(x)] [\bar{\psi}_{\nu_e}(x) \gamma^\mu (1 - \gamma^5) \psi_e(x)], \quad (5.1)$$

where $G_V = G_F \cos \vartheta_C$ with $G_F = 1.166 \times 10^{-5} \text{ MeV}^{-2}$ and ϑ_C are the Fermi weak coupling constant and the Cabibbo angle $\cos \vartheta_C = 0.975$ and $g_A = 1.267$; $\bar{\psi}_{\nu_e}(x)$ and $\psi_e(x)$ are the interpolating neutrino (anti-neutrino) and electron (positron) fields. The amplitude of $\bar{\nu}_e + D \rightarrow e^+ + n + n$ we obtain in the form

$$\begin{aligned} i\mathcal{M}(\bar{\nu}_e + D \rightarrow e^+ + n + n) &= -g_A M_N G_{\pi NN} \frac{G_V}{\sqrt{2}} \frac{3g_V}{2\pi^2} \frac{F_D(k^2)}{1 - \frac{1}{2} r_{nn} a_{nn} k^2 + i a_{nn} k} \\ &\quad \times e_\mu(Q) [\bar{v}(k_{\bar{\nu}_e}) \gamma^\mu (1 - \gamma^5) v(k_{e^+})] [\bar{u}(p_1) \gamma^5 u^c(p_2)], \end{aligned} \quad (5.2)$$

where the form factor $F_D(k^2)$ provides a spatial smearing of the deuteron. The factor $1/(1 - \frac{1}{2} r_{nn} a_{nn} k^2 + i a_{nn} k)$ describes the nn interaction in the final state which has been

taken into account by summing up one-nucleon loop diagrams, evaluated in leading order in the large N_C expansion, and renormalizing the wave functions of the neutrons. Since we work in the isotopical limit, we set $a_{nn} = a_{np} = -23.75$ fm and $r_{nn} = r_{np} = 2.75$ fm. The recent experimental values of the S-wave scattering length and the effective range of low-energy elastic nn scattering are equal to $a_{nn} = (-18.8 \pm 0.3)$ fm and $r_{nn} = (2.75 \pm 0.11)$ fm [31,32].

The amplitude Eq. (5.2), squared, averaged over polarizations of the deuteron and summed over polarizations of the final particles, reads

$$|\overline{\mathcal{M}(\bar{\nu}_e + D \rightarrow e^+ + n + n)}|^2 = \frac{144}{\pi^2} \frac{Q_D g_A^2 G_V^2 G_{\pi NN}^2 M_N^6 F_D^2(k^2)}{\left(1 - \frac{1}{2} r_{nn} a_{nn} k^2\right)^2 + a_{nn}^2 k^2} \left(E_{e^+} E_{\bar{\nu}_e} - \frac{1}{3} \vec{k}_{e^+} \cdot \vec{k}_{\bar{\nu}_e}\right). \quad (5.3)$$

In the RFMD the momentum dependence of the amplitude of the anti-neutrino disintegration of the deuteron agrees with that obtained in the PMA [33]. A much more complicated momentum dependence given in terms of the phenomenological form factors has been suggested by Mintz [34].

The cross section for the process $\bar{\nu}_e + D \rightarrow e^+ + n + n$ is defined by

$$\sigma_{cc}^{\bar{\nu}_e D}(E_{\bar{\nu}_e}) = \frac{1}{4E_D E_{\bar{\nu}_e}} \int |\overline{\mathcal{M}(\bar{\nu}_e + D \rightarrow e^+ + n + n)}|^2 \frac{1}{2} (2\pi)^4 \delta^{(4)}(Q + k_{\bar{\nu}_e} - p_1 - p_2 - k_{e^+}) \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 k_{e^+}}{(2\pi)^3 2E_{e^+}}, \quad (5.4)$$

where E_D , $E_{\bar{\nu}_e}$, E_1 , E_2 and E_{e^+} are the energies of the deuteron, the anti-neutrino, the neutrons and the positron. The abbreviation (cc) denotes the charged current. The integration over the phase volume of the (nne^+) -state we perform in the non-relativistic limit and in the rest frame of the deuteron

$$\begin{aligned} & \frac{1}{2} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 k_{e^+}}{(2\pi)^3 2E_{e^+}} (2\pi)^4 \delta^{(4)}(Q + k_{\bar{\nu}_e} - p_1 - p_2 - k_{e^+}) \\ & \times \frac{\left(E_{e^+} E_{\bar{\nu}_e} - \frac{1}{3} \vec{k}_{e^+} \cdot \vec{k}_{\bar{\nu}_e}\right) F_D^2(M_N T_{nn})}{\left(1 - \frac{1}{2} r_{nn} a_{nn} M_N T_{nn}\right)^2 + a_{nn}^2 M_N T_{nn}} = \frac{E_{\bar{\nu}_e} M_N^3}{1024 \pi^2} \left(\frac{E_{th}}{M_N}\right)^{7/2} \left(\frac{2m_e}{E_{th}}\right)^{3/2} \frac{8}{\pi E_{th}^2} \\ & \times \iint dT_{e^+} dT_{nn} \frac{\sqrt{T_{e^+} T_{nn}} F_D^2(M_N T_{nn})}{\left(1 - \frac{1}{2} r_{nn} a_{nn} M_N T_{nn}\right)^2 + a_{nn}^2 M_N T_{nn}} \\ & \times \left(1 + \frac{T_{e^+}}{m_e}\right) \sqrt{1 + \frac{T_{e^+}}{2m_e}} \delta(E_{\bar{\nu}_e} - E_{th} - T_{e^+} - T_{nn}) = \\ & = \frac{E_{\bar{\nu}_e} M_N^3}{1024 \pi^2} \left(\frac{E_{th}}{M_N}\right)^{7/2} \left(\frac{2m_e}{E_{th}}\right)^{3/2} \left(\frac{E_{\bar{\nu}_e}}{E_{th}} - 1\right)^2 f\left(\frac{E_{\bar{\nu}_e}}{E_{th}}\right), \end{aligned} \quad (5.5)$$

where T_{nn} is the kinetic energy of the nn system, T_{e^+} and $m_e = 0.511$ MeV are the kinetic energy and the mass of the positron, E_{th} is the anti-neutrino energy threshold of the reaction $\bar{\nu}_e + D \rightarrow e^+ + n + n$: $E_{th} = \varepsilon_D + m_e + (M_n - M_p) = (2.225 + 0.511 + 1.293)$ MeV =

4.029 MeV. The function $f(y)$, where $y = E_{\bar{\nu}_e}/E_{\text{th}}$, is defined as

$$f(y) = \frac{8}{\pi} \int_0^1 dx \frac{\sqrt{x(1-x)} F_D^2(M_N E_{\text{th}}(y-1)x)}{\left(1 - \frac{1}{2} r_{\text{nn}} a_{\text{nn}} M_N E_{\text{th}}(y-1)x\right)^2 + a_{\text{nn}}^2 M_N E_{\text{th}}(y-1)x} \times \left(1 + \frac{E_{\text{th}}}{m_e}(y-1)(1-x)\right) \sqrt{1 + \frac{E_{\text{th}}}{2m_e}(y-1)(1-x)}, \quad (5.6)$$

where we have changed the variable $T_{\text{nn}} = (E_{\bar{\nu}_e} - E_{\text{th}})x$. The function $f(y)$ is normalized to unity at $y = 1$, i.e. at threshold $E_{\bar{\nu}_e} = E_{\text{th}}$. Thus, the cross section for the anti-neutrino disintegration of the deuteron reads

$$\sigma_{\text{cc}}^{\bar{\nu}_e \text{D}}(E_{\bar{\nu}_e}) = \sigma_0 (y-1)^2 f(y), \quad (5.7)$$

where σ_0 is given by

$$\sigma_0 = Q_D G_{\pi \text{NN}}^2 \frac{9g_A^2 G_V^2 M_N^8}{512\pi^4} \left(\frac{E_{\text{th}}}{M_N}\right)^{7/2} \left(\frac{2m_e}{E_{\text{th}}}\right)^{3/2} = 4.58 \times 10^{-43} \text{ cm}^2. \quad (5.8)$$

The value $\sigma_0 = 4.58 \times 10^{-43} \text{ cm}^2$ agrees with the value $\sigma_0 = 4.68 \times 10^{-43} \text{ cm}^2$ obtained in the PMA [33] (see Fig. 7 of Ref. [13]).

The experimental data on the anti-neutrino disintegration of the deuteron are given in terms of the cross section averaged over the anti-neutrino energy spectrum [14]:

$$\langle \sigma_{\text{cc}}^{\bar{\nu}_e \text{D}}(E_{\bar{\nu}_e}) \rangle_{\text{exp}} = (9.83 \pm 2.04) \times 10^{-45} \text{ cm}^2.$$

In order to average the theoretical cross section Eq.(5.7) over the anti-neutrino spectrum we should use the spectrum given by Table YII of Ref.[14]. This yields

$$\langle \sigma_{\text{cc}}^{\bar{\nu}_e \text{D}}(E_{\bar{\nu}_e}) \rangle = 11.7 \times 10^{-45} \text{ cm}^2. \quad (5.9)$$

The theoretical value Eq. (5.9) agrees good with the experimental one $\langle \sigma_{\text{cc}}^{\bar{\nu}_e \text{D}}(E_{\bar{\nu}_e}) \rangle_{\text{exp}} = (9.83 \pm 2.04) \times 10^{-45} \text{ cm}^2$ [14].

6 Neutrino and anti-neutrino disintegration of the deuteron via neutral weak current interaction

The amplitude of the neutrino disintegration of the deuteron caused by neutral weak current $\nu_e + \text{D} \rightarrow \nu_e + \text{n} + \text{p}$ can be evaluated by analogy with the amplitude of the reaction $\bar{\nu}_e + \text{D} \rightarrow \text{e}^+ + \text{n} + \text{n}$ through one-nucleon loop exchanges (see Ref.[6]) and in leading order in the large N_C expansion [1]:

$$i\mathcal{M}(\nu_e + \text{D} \rightarrow \nu_e + \text{n} + \text{p}) = -g_A M_N \frac{G_F}{\sqrt{2}} \frac{3g_V}{4\pi^2} \frac{G_{\pi \text{NN}} F_D(k^2)}{1 - \frac{1}{2} r_{\text{np}} a_{\text{np}} k^2 + i a_{\text{np}} k} \times e_\mu(k_D) [\bar{u}(k'_{\nu_e}) \gamma^\mu (1 - \gamma^5) u(k_{\nu_e})] [\bar{u}(p_1) \gamma^5 u^c(p_2)], \quad (6.1)$$

where $\bar{u}(k'_{\nu_e})$, $u(k_{\nu_e})$, $\bar{u}(p_1)$ and $u^c(p_2)$ are the Dirac bispinors of the initial and the final neutrinos, and the nucleons. Then, the form factor $F_D(k^2)$ provides a spatial smearing of

the deuteron and the factor $1/(1 - \frac{1}{2}r_{\text{np}}a_{\text{np}}k^2 + i a_{\text{np}} k)$ describes the np interaction in the final state.

The amplitude Eq.(6.1) squared, averaged over polarizations of the deuteron, summed over polarizations of the nucleons reads

$$|\overline{\mathcal{M}(\nu_e + \text{D} \rightarrow \nu_e + \text{n} + \text{p})}|^2 = \frac{36}{\pi^2} \frac{Q_{\text{D}} g_{\text{A}}^2 G_{\text{F}}^2 G_{\pi\text{NN}}^2 M_{\text{N}}^6 F_{\text{D}}^2(k^2)}{\left(1 - \frac{1}{2}r_{\text{np}}a_{\text{np}}k^2\right)^2 + a_{\text{np}}^2 k^2} \left(E'_{\nu_e} E_{\nu_e} - \frac{1}{3}\vec{k}'_{\nu_e} \cdot \vec{k}_{\nu_e}\right). \quad (6.2)$$

In the rest frame of the deuteron the cross section for the process $\nu_e + \text{D} \rightarrow \nu_e + \text{n} + \text{p}$ is defined

$$\sigma_{\text{nc}}^{\nu_e \text{D}}(E_{\bar{\nu}_e}) = \frac{1}{4M_{\text{D}}E_{\nu_e}} \int |\overline{\mathcal{M}(\nu_e + \text{D} \rightarrow \nu_e + \text{n} + \text{p})}|^2 (2\pi)^4 \delta^{(4)}(k_{\text{D}} + k_{\nu_e} - p_1 - p_2 - k'_{\nu_e}) \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 k'_{\nu_e}}{(2\pi)^3 2E'_{\nu_e}}. \quad (6.3)$$

The abbreviation (nc) denotes the neutral current. The integration over the phase volume of the $(\text{np}\nu_e)$ -state we perform in the non-relativistic limit and in the rest frame of the deuteron,

$$\begin{aligned} & \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 k'_{\nu_e}}{(2\pi)^3 2E'_{\nu_e}} (2\pi)^4 \delta^{(4)}(k_{\text{D}} + k_{\nu_e} - p_1 - p_2 - k'_{\nu_e}) \\ & \left(E_{\nu_e} E'_{\nu_e} - \frac{1}{3}\vec{k}_{\nu_e} \cdot \vec{k}'_{\nu_e}\right) \frac{F_{\text{D}}^2(M_{\text{N}}T_{\text{np}})}{\left(1 - \frac{1}{2}r_{\text{np}}a_{\text{np}}M_{\text{N}}T_{\text{np}}\right)^2 + a_{\text{np}}^2 M_{\text{N}}T_{\text{np}}} = \\ & = \frac{E_{\nu_e} M_{\text{N}}^3}{210\pi^3} \left(\frac{E_{\text{th}}}{M_{\text{N}}}\right)^{7/2} (y-1)^{7/2} \Omega_{\text{np}\nu_e}(y). \end{aligned} \quad (6.4)$$

The function $\Omega_{\text{np}\nu_e}(y)$, where $y = E_{\bar{\nu}_e}/E_{\text{th}}$ and $E_{\text{th}} = \varepsilon_{\text{D}} = 2.225 \text{ MeV}$ is threshold of the reaction, is defined as

$$\Omega_{\text{np}\nu_e}(y) = \frac{105}{16} \int_0^1 dx \frac{\sqrt{x}(1-x)^2}{\left(1 - \frac{1}{2}\frac{r_{\text{np}}a_{\text{np}}}{r_{\text{D}}^2}(y-1)x\right)^2 + \frac{a_{\text{np}}^2}{r_{\text{D}}^2}(y-1)x} \frac{1}{(1+(y-1)x)^2}, \quad (6.5)$$

where we have changed the variable $T_{\text{np}} = (E_{\bar{\nu}_e} - E_{\text{th}})x$ and used the relation $M_{\text{N}}E_{\text{th}} = 1/r_{\text{D}}^2$ at $E_{\text{th}} = \varepsilon_{\text{D}}$. The function $\Omega_{\text{np}\nu_e}(y)$ is normalized to unity at $y = 1$, i.e., at threshold $E_{\bar{\nu}_e} = E_{\text{th}}$.

The cross section for the neutrino disintegration of the deuteron caused by the neutral weak current $\nu_e + \text{D} \rightarrow \nu_e + \text{n} + \text{p}$ reads

$$\sigma_{\text{nc}}^{\nu_e \text{D}}(E_{\nu_e}) = \sigma_0 (y-1)^{7/2} \Omega_{\text{np}\nu_e}(y), \quad (6.6)$$

where σ_0 is defined by

$$\sigma_0 = Q_{\text{D}} G_{\pi\text{NN}}^2 \frac{3g_{\text{A}}^2 G_{\text{F}}^2 M_{\text{N}}^8}{140\pi^5} \left(\frac{E_{\text{th}}}{M_{\text{N}}}\right)^{7/2} = 1.84 \times 10^{-43} \text{ cm}^2. \quad (6.7)$$

In our approach the cross section for the disintegration of the deuteron by neutrinos $\nu_e + D \rightarrow \nu_e + n + p$ coincides with the cross section for the disintegration of the deuteron by anti-neutrinos $\bar{\nu}_e + D \rightarrow \bar{\nu}_e + n + p$, $\sigma_{\text{nc}}^{\nu_e D}(E_{\nu_e}) = \sigma_{\text{nc}}^{\bar{\nu}_e D}(E_{\bar{\nu}_e})$. Therefore, we compare our results with the experimental data on the disintegration of the deuteron by anti-neutrinos [14]. The experimental value of the cross section for the anti-neutrino disintegration of the deuteron $\bar{\nu}_e + D \rightarrow \bar{\nu}_e + n + p$ averaged over the anti-neutrino spectrum reads [14]: $\langle \sigma_{\text{nc}}^{\bar{\nu}_e D}(E_{\bar{\nu}_e}) \rangle_{\text{exp}} = (6.08 \pm 0.77) \times 10^{-45} \text{ cm}^2$.

By using the anti-neutrino spectrum given by Table YII of Ref.[14] for the calculation of the average value of the theoretical cross section Eq.(6.6) we obtain

$$\langle \sigma_{\text{nc}}^{\bar{\nu}_e D}(E_{\bar{\nu}_e}) \rangle = 6.4 \times 10^{-45} \text{ cm}^2. \quad (6.8)$$

The theoretical value Eq. (6.8) agrees good with the experimental one $\langle \sigma_{\text{nc}}^{\bar{\nu}_e D}(E_{\bar{\nu}_e}) \rangle_{\text{exp}} = (6.08 \pm 0.77) \times 10^{-45} \text{ cm}^2$ [14].

7 Conclusion

The main goal of the paper is to show that: 1) Chiral perturbation theory can be incorporated in the RFMD and 2) the amplitudes of low-energy elastic nucleon-nucleon scattering contributing to the reactions of the photo-magnetic and anti-neutrino disintegration of the deuteron can be described in the RFMD in agreement with low-energy nuclear phenomenology.

By example of the evaluation of the amplitude for the radiative M1-capture $n + p \rightarrow D + \gamma$ we have shown that Chiral perturbation theory can be incorporated into the RFMD. We have considered chiral meson-loop corrections from the virtual meson transitions $\pi \rightarrow a_1 \gamma$, $a_1 \rightarrow \pi \gamma$, $\pi \rightarrow (\omega, \rho) \gamma$, $(\omega, \rho) \rightarrow \pi \gamma$, $\sigma \rightarrow (\omega, \rho) \gamma$ and $(\omega, \rho) \rightarrow \sigma \gamma$, where σ is a scalar partner of pions under chiral $SU(2) \times SU(2)$ transformations in $(\text{CHPT})_q$ with a linear realization of chiral $U(3) \times U(3)$ symmetry. These virtual meson transitions give contributions to the effective interactions of nucleons $\delta \mathcal{L}_{\text{eff}}^{\text{NN}\gamma}(x)$ coupled to a magnetic field

$$\delta \mathcal{L}_{\text{eff}}^{\text{NN}\gamma}(x) = \frac{ie}{4M_N} \mu_N^{(\chi)} \bar{N}(x) \sigma_{\mu\nu} N(x) F^{\mu\nu}(x). \quad (7.1)$$

The effective magnetic moments $\mu_N^{(\chi)}$, caused by chiral meson-loop corrections, have been evaluated in leading order in the large N_C expansion [1] and renormalized according to the renormalization procedure developed in $(\text{CHPT})_q$ for the evaluation of chiral meson-loop corrections (see *Ivanov* in Refs. [9]). Since the renormalized expressions should vanish in the chiral limit $M_\pi \rightarrow 0$, the contributions of the virtual meson transitions with intermediate σ -meson, finite in the chiral limit, have been subtracted [9]. Such a cancellation of the σ -meson contributions in the one-meson loop approximation agrees with Chiral perturbation theory using a non-linear realization of chiral symmetry, where σ -meson like exchanges can appear only in two-meson loop approximation. The non-trivial contributions vanishing in the chiral limit have been obtained only from the virtual meson transitions with intermediate π -meson.

The numerical value of the cross section for the M1-capture accounting for the contributions of chiral one-meson loop corrections amounts to $\sigma^{\text{np}}(k) = 287.2 \text{ mb}$. This

value differs from the experimental one $\sigma^{\text{np}}(k)_{\text{exp}} = (334.2 \pm 0.5) \text{ mb}$ by about 14%. For the description of this discrepancy we have taken into account the contribution of the $\Delta(1232)$ resonance. The total cross section for the neutron–proton radiative capture has been found dependent on the parameter Z defining the $\pi\Delta\text{N}$ coupling off-mass shell of the $\Delta(1232)$ resonance: $\sigma^{\text{np}}(k) = 287.2 (1 + 0.64 (1 - 2Z))^2 \text{ mb}$ Eq.(3.23). In order to fit the experimental value of the cross section we should set $Z = 0.438$. This agrees with the experimental bound $|Z| \leq 1/2$ [23].

When matching our result for the cross section for the M1–capture with the recent one obtained in the EFT approach by Chen, Rupak and Savage [35]: $\sigma^{\text{np}}(k) = (287.1 + 6.51 \text{ } ^\# L_1) \text{ mb}$ (see Eq.(3.49) of Ref. [35]), we accentuate the dependence of the cross section on the parameter $^\# L_1$ which has been fixed from the experimental data. Since the contribution of the $\Delta(1232)$ resonance has not been considered in Ref.[35] and there is no uncertainties related to the parameter Z , $^\# L_1$ is a free parameter of the approach. Unlike Ref.[35] the cross section calculated in the RFMD does not contain free parameters.

The cross section for the photo–magnetic disintegration of the deuteron $\gamma + \text{D} \rightarrow \text{n} + \text{p}$ has been evaluated for energies far from threshold. For this aim we have taken into account the np interaction in the final state by summing up an infinite series of one–nucleon loop diagrams which have been calculated in leading order in the large N_C expansion. This has given the amplitude of low–energy elastic np scattering contributing to the amplitude of $\gamma + \text{D} \rightarrow \text{n} + \text{p}$ in the form agreeing with low–energy nuclear phenomenology, i.e. defined by the S–wave scattering length a_{np} and the effective range r_{np} . This result relaxes substantially the statement by Bahcall and Kamionkowski [18] that in the RFMD due to the effective local four–nucleon interaction Eq.(1.1) one cannot describe low–energy elastic NN scattering in agreement with low–energy nuclear phenomenology. Nevertheless, the problem of the description of low–energy elastic pp scattering accounting for the Coulomb repulsion still remains.

We have shown that the dependence of the amplitude of the photo–magnetic disintegration of the deuteron on the deuteron radius r_{D} in the form of the factor $1/(1 + r_{\text{D}}^2 k^2)$ can be justified in the RFMD by means of the direct inclusion of the wave functions of the deuteron and the np–pair in the $^1\text{S}_0$ –state. However, such an inclusion leads to significant complexification of the model consideration of which goes beyond the scope of this paper. The problem of the factor $1/(1 + r_{\text{D}}^2 k^2)$, universal for all low–energy processes of the deuteron coupled to the NN system in the $^1\text{S}_0$ –state, can be preferably solved phenomenologically. Referring to the possibility to derive this factor in the RFMD by the way having been discussed in Sect. 3 we have suggested to multiply the amplitudes of low–energy nuclear transitions evaluated near thresholds by a factor $1/(1 + r_{\text{D}}^2 k^2)$. In other words, we have introduced an universal form factor $F_{\text{D}}(k^2) = 1/(1 + r_{\text{D}}^2 k^2)$ describing a spatial smearing of the deuteron coupled to the NN system in the $^1\text{S}_0$ –state.

This procedure has been applied to the evaluation of the cross sections for the anti–neutrino disintegration of the deuteron caused by charged $\bar{\nu}_e + \text{D} \rightarrow \text{e}^+ + \text{n} + \text{n}$ and neutral $\bar{\nu}_e + \text{D} \rightarrow \bar{\nu}_e + \text{n} + \text{p}$ weak currents. The theoretical cross sections averaged over the anti–neutrino spectrum $\langle \sigma_{\text{cc}}^{\bar{\nu}_e \text{D}}(E_{\bar{\nu}_e}) \rangle = 11.7 \times 10^{-45} \text{ cm}^2$ and $\langle \sigma_{\text{nc}}^{\bar{\nu}_e \text{D}}(E_{\bar{\nu}_e}) \rangle = 6.4 \times 10^{-45} \text{ cm}^2$ agree good with recent experimental data $\langle \sigma_{\text{cc}}^{\bar{\nu}_e \text{D}}(E_{\bar{\nu}_e}) \rangle_{\text{exp}} = (9.83 \pm 2.04) \times 10^{-45} \text{ cm}^2$ and $\langle \sigma_{\text{nc}}^{\bar{\nu}_e \text{D}}(E_{\bar{\nu}_e}) \rangle_{\text{exp}} = (6.08 \pm 0.77) \times 10^{-45} \text{ cm}^2$ obtained by the Reines’s experimental group [14].

The cross sections for the reactions $\bar{\nu}_e + D \rightarrow e^+ + n + n$ and $\bar{\nu}_e + D \rightarrow \bar{\nu}_e + n + p$ have been recently calculated by Butler and Chen [36] in the EFT approach. The obtained results have been written in the following general form $\sigma = (a + b L_{1,A}) \times 10^{-42} \text{ cm}^2$ (see Table I of Ref. [36]), where a and b are the parameters which have been calculated in the approach, whereas $L_{1,A}$ is a free one. Thus, unlike the cross sections given by Eqs.(5.7) and (6.6), where there are no free parameters, the cross sections for the anti-neutrino disintegration of the deuteron [36] as well as for the neutron-proton radiative capture [35] calculated in the EFT approach depend on free parameters. Due to independence of the cross sections Eqs.(5.7) and (6.6) on free parameters we can analyse and value in the RFMD not only chiral meson-loop corrections but the corrections mentioned recently by Vogel and Beacom [37].

Acknowledgement

We are grateful to Prof. Kamionkowski for helpful remarks and encouragement for further applications of the expounded in the paper technique to the evaluation of the astrophysical factor for pp fusion and the cross section for the neutrino disintegration of the deuteron $\nu_e + D \rightarrow e^- + p + p$ by accounting for the Coulomb repulsion between the protons.

We thank Prof. Beacom for calling our attention to the experimental data [14]. Discussions of the experimental data with Prof. Sobel and Prof. Price are greatly appreciated.

References

- [1] A. N. Ivanov, H. Oberhummer, N. I. Troitskaya and M. Faber, *The relativistic field theory model of the deuteron from low-energy QCD*, nucl-th/9908029, August 1999.
- [2] A. N. Ivanov, N. I. Troitskaya, M. Faber and H. Oberhummer, Phys. Lett. B361 (1995) 74.
- [3] A. N. Ivanov, N. I. Troitskaya, M. Faber and H. Oberhummer, Nucl. Phys. A617 (1997) 414 and references therein.
- [4] A. N. Ivanov, N. I. Troitskaya, M. Faber and H. Oberhummer, Nucl. Phys. A625 (1997) 896 (Erratum).
- [5] A. N. Ivanov, H. Oberhummer, N. I. Troitskaya and M. Faber, *Solar proton burning, photon and anti-neutrino disintegration of the deuteron in the relativistic field theory model of the deuteron*, nucl-th/9810065, October 1998.
- [6] A. N. Ivanov, H. Oberhummer, N. I. Troitskaya and M. Faber, *Solar neutrino processes in the relativistic field theory model of the deuteron*, nucl-th/9811012, November 1998.
- [7] A. E. Cox, A. R. Wynchank and C. H. Collie, Nucl. Phys. 74 (1965) 497 and references therein.
- [8] T.-S. Park, D.-P. Min and M. Rho, Phys. Rev. Lett. 74 (1995) 4153; Nucl. Phys. A596 (1996) 515; T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, *Effective field theory approach to $n(\text{polarized}) + p(\text{polarized}) \rightarrow D + \gamma$ at threshold*, nucl-th/9906005, June 1999.
- [9] A. N. Ivanov, M. Nagy and N. I. Troitskaya, Int. J. Mod. Phys. A7 (1992) 7305; A. N. Ivanov, Int. J. Mod. Phys. A8 (1993) 853; A. N. Ivanov, N. I. Troitskaya and M. Nagy, Int. J. Mod. Phys. A8 (1993) 2027; *ibid.* A8 (1993) 3425; Phys. Lett. B308 (1993) 111; *ibid.* B295 (1992) 308; A. N. Ivanov and N. I. Troitskaya, Nuovo Cim. A108 (1995) 555; Phys. Lett. B342 (1995) 323; *ibid.* B387 (1996) 386; *ibid.* B388 (1996) 869 (Erratum); *ibid.* B390 (1997) 341; F. Hussain, A. N. Ivanov and N. I. Troitskaya, Phys. Lett. B348 (1995) 609; *ibid.* B369 (1996) 351.
- [10] A. N. Ivanov, M. Nagy and N. I. Troitskaya, Phys. Rev. C59 (1999) 451; Ya. A. Berdnikov, A. N. Ivanov, V. F. Kosmach and N. I. Troitskaya, Phys. Rev. C60 (1999) 015201.
- [11] D. O. Riska and G. E. Brown, Phys. Lett. B38 (1972) 193.
- [12] A. N. Ivanov, N. I. Troitskaya, H. Oberhummer and M. Faber, Z. Phys. A358 (1997) 81.
- [13] T. L. Jenkins, F. E. Kinard and F. Reines, Phys. Rev. 185 (1969) 1599.
- [14] S. P. Riley, Z. D. Greenwood, W. R. Kroop, L. R. Price, F. Reines, H. W. Sobel, Y. Declais, A. Etenko and M. Skorokhvatov, Phys. Rev. C59 (1999) 1780.

- [15] S. Weinberg, Phys. Lett. B251 (1990) 288; Nucl. Phys. B363 (1991) 3; Phys. Lett. B295 (1992) 114.
- [16] D. R. Kaplan, M. J. Savage and M. B. Wise, Nucl. Phys. B478 (1996) 629 and references therein; S. R. Beane, T. D. Cohen and D. R. Phillips, Nucl. Phys. A632 (1998) 445; T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, Nucl. Phys. A646 (1999) 83.
- [17] S. DeBenedetti, in *NUCLEAR INTERACTIONS*, John Wiley & Sons, Inc., New York, 1964 and references therein.
- [18] J. N. Bahcall and M. Kamionkowski, Nucl. Phys. A625 (1997) 893.
- [19] C. Caso *et al.*, Eur. Phys. J. C3 (1998) 1.
- [20] W. Rarita and J. Schwinger, Phys. Rev. 60 (1941) 61.
- [21] L. M. Nath, B. Etemadi and J. D. Kimel, Phys. Rev. D3 (1971) 2153.
- [22] J. Kambor, *The $\Delta(1232)$ as an Effective Degree of Freedom in Chiral Perturbation Theory*, Talk given at the Workshop on Chiral Dynamics, 1997 Mainz, Germany, September 1–5, 1997; hep-ph/9711484 26 November 1997.
- [23] K. Kabir, T. K. Dutta, Muslema Pervin and L. M. Nath, *The Role of $\Delta(1232)$ in Two-pion Exchange Three-nucleon Potential*, hep-th/9910043, October 1999.
- [24] V. Bernard, N. Kaiser and Ulf-G. Meissner, Int. J. Mod. Phys. E4 (1995) 193 and references therein.
- [25] R. D. Peccei, Phys. Rev. 181 (1969) 1902 and references therein.
- [26] M. G. Olsson and E. T. Osypowski, Nucl. Phys. B87 (1975) 399.
- [27] M. M. Nagels *et al.*, Nucl. Phys. B147 (1979) 253.
- [28] I. S. Gertsein and R. Jackiw, Phys. Rev. 181 (1969) 1955.
- [29] A. Bohr and B. R. Mottelson, in *NUCLEAR STRUCTURE*, Vol. I, W. A. Benjamin, Inc., New York, 1969.
- [30] C. Itzykson and J.-B. Zuber, in *QUANTUM FIELD THEORY*, McGraw-Hill Book Company, New York, 1980, p. 217.
- [31] G. A. Miller, B. M. K. Nefkens and I. Slaus, Phys. Rep. 194 (1990) 1; G. A. Miller and W. N. T. van Oers, in *Symmetries and Fundamental Interactions in Nuclei*, W. C. Haxton and E. M. Henley, eds., World Scientific Singapore, 1995, p.127.
- [32] R. Machleidt and M. K. Banerjee, *Charge-dependence of the π NN coupling constant and charge-dependence of the NN interaction*, nucl-th/9908066, August 1999.

- [33] J. Weneser, Phys. Rev. 105 (1957) 1335; T. Ahrens and T. P. Lang, Phys. Rev. C3 (1971) 979; J. Hošek and E. Truhlik, Phys. Rev. C23 (1981) 665; F. T. Avignone, Phys. Rev. D24 (1981) 778.
- [34] S. L. Mintz, Phys. Rev. C23 (1981) 421, *ibid.* C24 (1981) 1799.
- [35] J.-W. Chen, G. Rupak and M. J. Savage, Nucl. Phys. A653 (1999) 386.
- [36] M. Butler and J.-W. Chen, *Elastic and Inelastic neutrino-Deuteron Scattering in Effective Field Theory*, nucl-th/9905059, June 1999.
- [37] P. Vogel and J. F. Beacom, Phys. Rev. D60 (1999) 053003.